The Ghost of Pauli: Implications of Energy Conservation in Neutrino Oscillations

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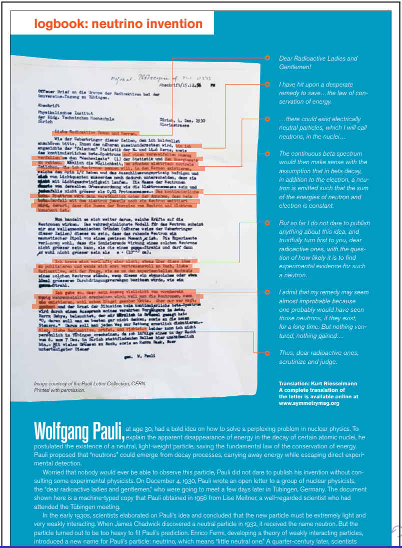
Based on: "Neutrino oscillations: Inevitability of non-standard interactions or a sterile neutrino, D. V. Ahluwalia, D. Schritt [arXiv:0911.2965 hep-ph]"

Pauli's Letter dated 04 December 1930

β decay/to save energy conservation and other elements of physics/Pauli infers

- Existence of neutrinos[a]
- Neutrinos are massive
- Neutrinos have spin $1/2$[b]
- Neutrinos must interact weakly[c]

(Reproduced from Symmetry, March 2007)



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[a] Initially neutrons, later neutrinos by Fermi at a suggestion of Majorana.

[b] From the "wrong" statistics of N and Li-6 nuclei.

[c] To reconcile with the null results of calorimetry experiments. C. D. Ellis, W. A. Wooster, Nature, **119**, 563-564 (1927). …"thick-walled calorimeter"!

Pauli's Neutrino

Pauli's neutrino is a mass eigenstate

$$|\text{Pauli's neutrino}\rangle = |m\rangle$$

Its time evolution in the matter-free region is given by

$$|\text{Pauli's neutrino}\rangle \xrightarrow{t>0} \exp\left(\frac{-iEt}{\hbar}\right)|m\rangle$$

Pauli's neutrino, with $m = 0$ (and with additional flavours) is the neutrino of the standard model of HEP.

Despite spectacular success, when confronted with data on solar neutrinos, atmospheric neutrinos, and other reactor and accelerator experiments, Pauli's neutrinos encounter insurmountable problems.

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Pontecorvo's Neutrino

The anomalies and problems evaporate dramatically if the flavour eigenstates are assumed to be linear superposition of the Pauli's neutrinos. This suggestion originates with Bruno Pontecorvo. One thus introduces the Pontecorvo's neutrinos

$$|\nu_\ell\rangle = \sum_i U_{\ell i}|m_i\rangle$$

The flavour index $\ell = e, \mu\ \tau$, and the mass eigenstate index $i = 1, 2, 3$.
In general m_i are non-degenerate.
U is the 3×3 neutrino mixing matrix.
It is unitary.
For Dirac neutrinos it is defined by three independent angles and a CP-violating phase (which may or may not be zero).

Time evolution of Pontecorvo's Neutrinos

$$|\text{Pontecorvo's neutrino}\rangle \xrightarrow{t>0} \sum_i U_{\ell i} \underbrace{\exp\left(\frac{-iE_i t}{\hbar}\right)}_{\text{Introduces \underline{relative} phases between the three } |\mathrm{m_i}\rangle} |m_i\rangle$$

For relativistic neutrinos $E_i \approx p + m_i^2/(2p) \approx p + m_i^2/(2E_i)$. Standard neutrino oscillation phenomenology assumes that all three mass eigenstates have same linear momentum p. This assumption is easily relaxed without changing the relevant results. Now onwards $\hbar = 1$, $c = 1$.

$\Rightarrow$

Neutrino flavour oscillations as each of the mass eigenstate evolves with a different phase ($E_i \neq E_j$).

Flavour oscillation clocks: QM and GR

Flavour oscillations can thus be used to make flavour oscillation clocks. These clocks redshift precisely as required by GR.

D. V. Ahluwalia and C. Burgard, Gen. Rel. Grav.28:1161-1170,1996
D. V. Ahluwalia and C. Burgard, Phys. Rev. D57:4724-4727,1998.
For a several months there was at least one paper on the subject every week. Though these papers are no longer cited often, there are several hundred papers that have been inspired by this work.

The physics is simple: Each of the underlying mass eigenstate picks up a different gravitationally-induced phase. That a purely QM calculation, in a curved background, yields precisely the predictions of GR reflects certain universality and harmony of QM and GR.

Anything else?: The Ghost of Pauli

To understand physical implications of neutrino oscillations, e.g. for supernova explosions, one needs precise set of parameters that underlie flavour oscillations of neutrinos. Thus, the task of neutrino-oscillation data analysis is to determine

- the two independent mass squared differences $\Delta m^2_{21} := m^2_2 - m^2_1$ and $\Delta m^2_{32} := m^2_3 - m^2_2$
- three mixing angles
- the CP-violating phase

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{12} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

The matter-free oscillation parameters are modified my matter effects (MSW effect). For example, W-exchange induced ν_e forward-scattering from electrons in medium contribute $\sqrt{2}G_F N_e$ to the interaction energy.[1] The Z-induced forward scattering does not contribute to oscillations in absence of sterile neutrinos. This happens because it induces a global, and not a relative, phase.

Once MSW matter effects are taken into account, the current set of data yields

$$\Delta m^2_{21} = 7.59^{+0.19}_{-0.21} \times 10^{-5} eV^2$$
$$|\Delta m^2_{32}| = 2.43^{\pm}0.13 \times 10^{-3} eV^2$$
$$\sin^2(2\theta_{13}) < 0.19 \ (90\% CL) \ i.e., \theta_{13} \approx 0$$
$$\sin^2(2\theta_{23}) > 0.92 \ (90\% CL)$$
$$\sin^2(2\theta_{12}) = 0.87 \pm 0.03$$

Reference: C. Amsler et al. (Particle Data Group), Phys. Lett. B **667**, 1, 2008.

[1] Here G_F = Fermi constant, N_e = number of electrons per unit volume.

But, there are dark clouds on the horizon

Now there are several lingering problems on the neutrino front that are likely to throw the neutrino community in a disarray.

- One of these is the LSND (Los Alamos) and the ongoing MiniBOONE (Fermilab). Both require a new mass-squared difference of around 1 eV^2 ... Interesting things are happening at MiniBOONE. A fourth generation of neutrino (sterile/dark) seems inevitable.
- Latest observations on ultra-faint dwarf galaxy Willman 1, according to Lowenstein and Kusenko (arXiv:0912.0552), suggest a dark/sterile neutrino with sterile neutrino mass of 5 keV.
- The sterile neutrino of Lowenstein and Kusenko also resolves the pulsar kick problem.
- A sterile neutrino also has serious and positive implications for supernova explosions.

Anything else?: The Ghost of Pauli

Now consider an idealised beam of neutrinos, initially containing electron neutrinos only. The state of a single electron neutrino $|\nu_e\rangle = \sum_i U_{ei}|\nu_i\rangle$ evolves with time, and at any given time it has a definite probability of being found — on measurement — as one of the three neutrinos, $|\nu_\ell\rangle$. Unless a flavour measurement is made the energy expectation value remains constant in time. However, if a flavour measurement is made the initial energy content $\mathcal{E}(t=0) := N\langle E_e\rangle$, at any later time t gets distributed into three components

$$\mathcal{E}(t>0) = \sum_\ell \underbrace{NP(\nu_e \to \nu_\ell)\langle E_\ell\rangle}_{\text{contribution from flavour } \ell}$$

where N represents the number of electron neutrinos at the initial time $t=0$ and $P(\nu_e \to \nu_\ell)$ is the oscillation probability to flavour ℓ (at time t).

Anything else?: The Ghost of Pauli ...

For a general mixing matrix the indicated energy balance is violently violated.

To avoid this catastrophe, we conjecture that given a p, the three $\langle E_\ell \rangle$ must be flavour independent.

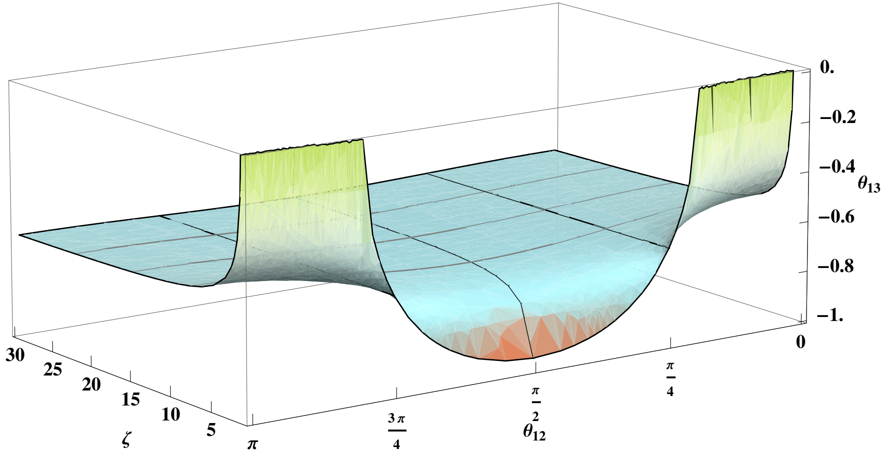
Accordingly, if we restrict the mixing matrix U such that all three $\langle E_\ell \rangle$ are equal, the conservation of energy requirement translates to a simple condition of unitarity: $\sum_\ell P(\nu_e \rightarrow \nu_\ell) = 1$.

Implementing the Conjecture

Conjecture yields two complicated constraints on the mixing matrix U. With some effort these can be solved for θ_{13} and ζ

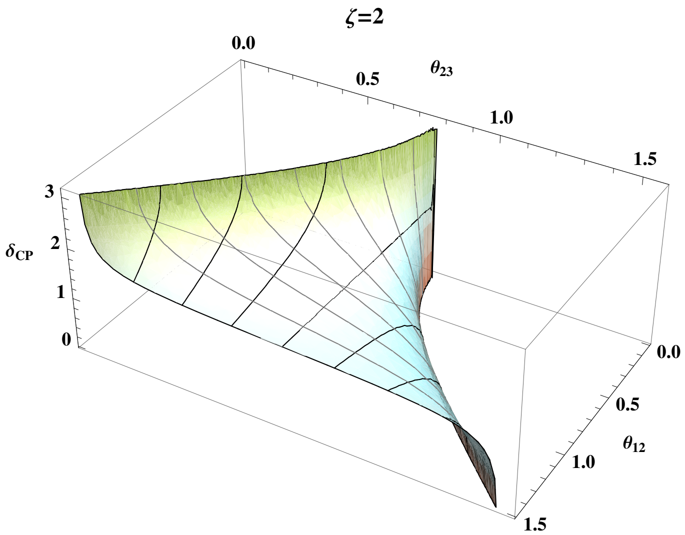
$$\zeta := -\frac{\Delta m_{32}^2}{\Delta m_{21}^2}$$

Thus out of six oscillation parameters only four are independent.



0.
−0.2
−0.4
−0.6
−0.8
−1.
θ_{13}
30
25
20
15
10
5
ζ
π
$\frac{3\pi}{4}$
$\frac{\pi}{2}$
θ_{12}
$\frac{\pi}{4}$
0

!!a



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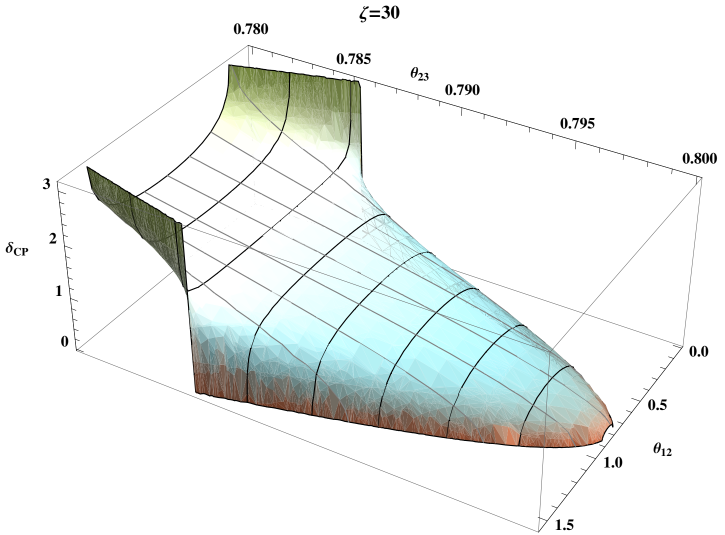

Conclusions

Considerations on energy conservation in neutrino oscillation phenomenology suggest that either there is at least one sterile neutrino or/and there are non-standard neutrino interactions. For the three flavour scheme examined here, the energy conservation constraint yields:

$$\theta_{13} = \eta_1 \arccos\left[\eta_2 \sqrt{\frac{2}{3}\left(\frac{2\zeta - 1}{2\zeta - \cos(2\theta_{12}) - 1}\right)}\right]$$

$$\delta_{CP} = \eta_3 \arccos\left[\eta_4 \Big(2\zeta + 3\cos(2\theta_{12}) - 1\Big)\frac{\cot(2\theta_{23})}{\sin(2\theta_{12})}\right.$$

$$\left.\times\sqrt{\frac{1}{3}\left(\frac{2\zeta - \cos(2\theta_{12}) - 1}{2\zeta - 3\cos(2\theta_{12}) - 1}\right)}\right]$$

where the η are either $+1$ or -1. Extension of this work to include sterile neutrinos is urgently needed.