Cosmic acceleration from structure formation

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Outline

1. Statement:

   volume-averaged expansion of space can accelerate even if the local expansion decelerates everywhere

2. Mathematical foundation

3. Physical interpretation

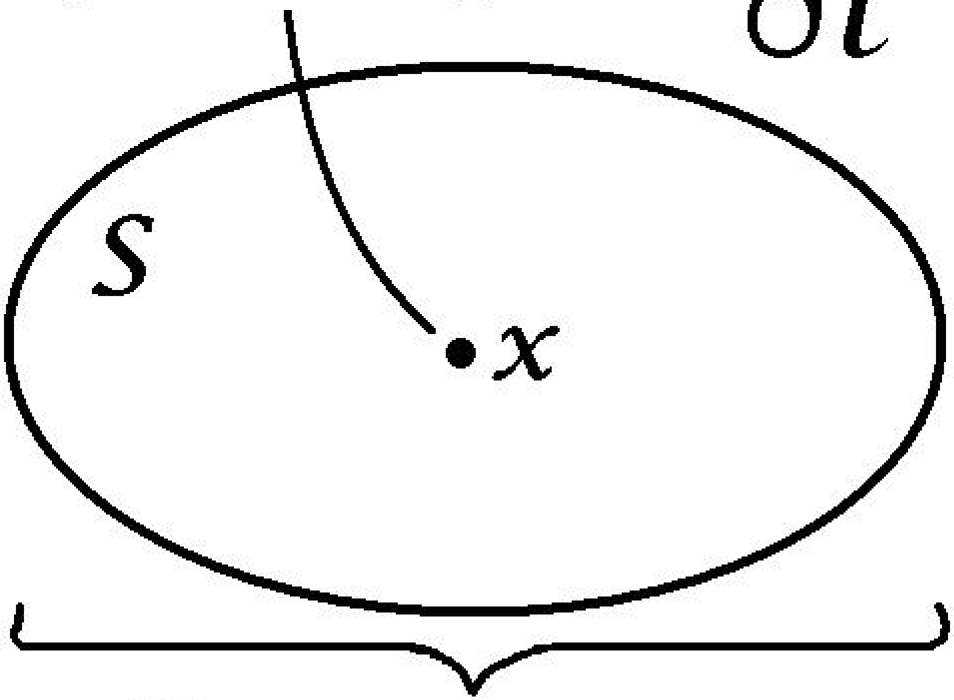
4. Observational status

5. Conclusions

This talk is based on General Relativity.

No new physics is proposed.

$$\forall \boldsymbol{x} \in \boldsymbol{S} : \frac{\partial}{\partial t}\theta < 0$$



$$\theta \equiv \frac{1}{\sqrt{\mathsf{g}}}\frac{\partial\sqrt{\mathsf{g}}}{\partial t}$$

$$\frac{\partial}{\partial t}\langle\theta\rangle_S > 0$$

Mathematical foundation

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- Take time-derivative of this:

$$\frac{\partial}{\partial t}\langle S(x,t)\rangle = \frac{\int[\sqrt{g}\partial_t S + S\partial_t(\sqrt{g})]d^3x}{\int \sqrt{g}d^3x} - \frac{\int S\sqrt{g}d^3x \int \partial_t(\sqrt{g})d^3x}{(\int \sqrt{g}d^3x)^2}$$

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$$\partial_t\langle S\rangle = \langle \partial_t S\rangle + \langle S\theta\rangle - \langle S\rangle\langle\theta\rangle$$

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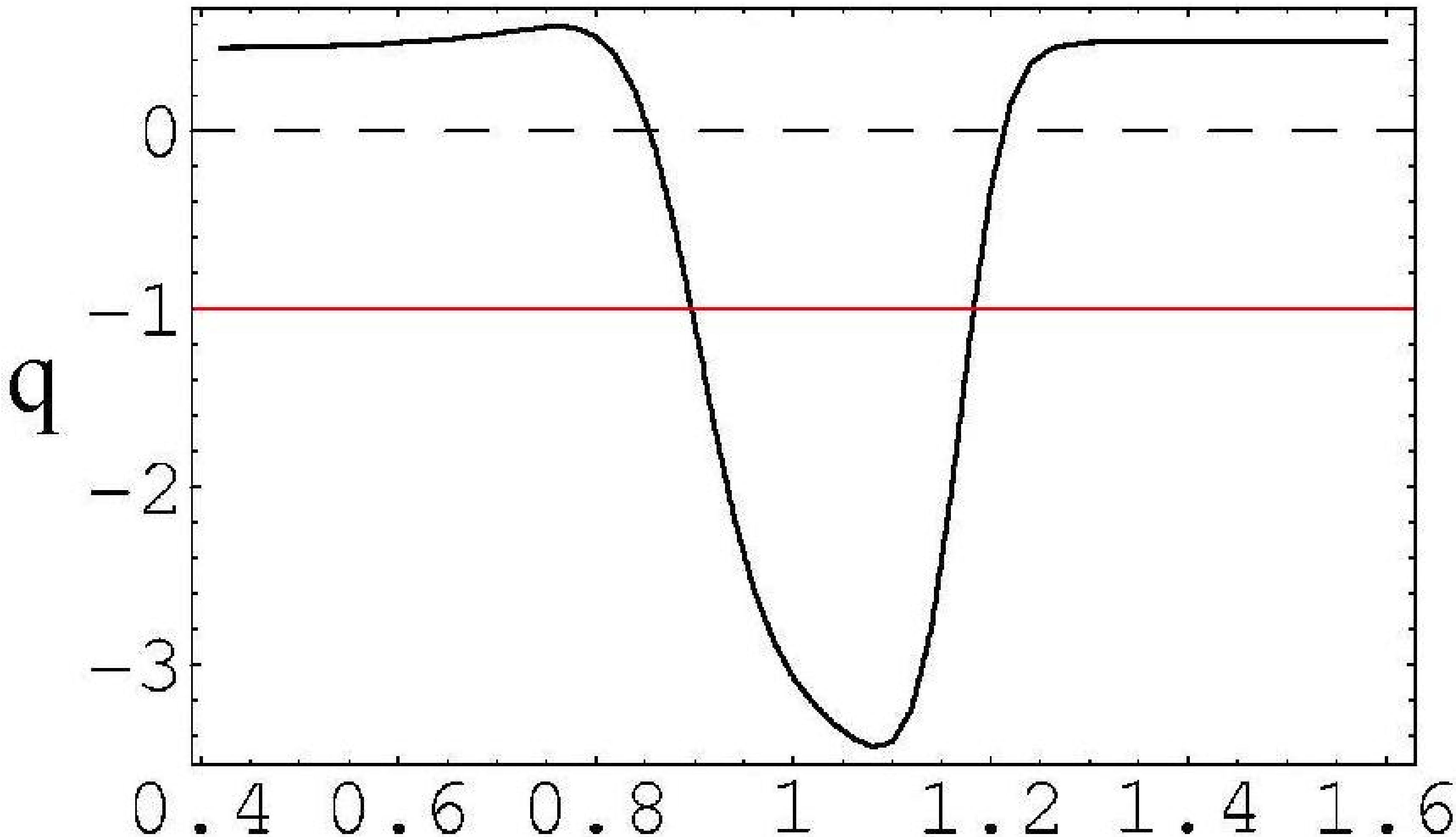
- For large variance of the expansion rate, the right hand side can be positive

Numerical proof by Chuang, Gu and Hwang using an exact LTB solution



Physical interpretation

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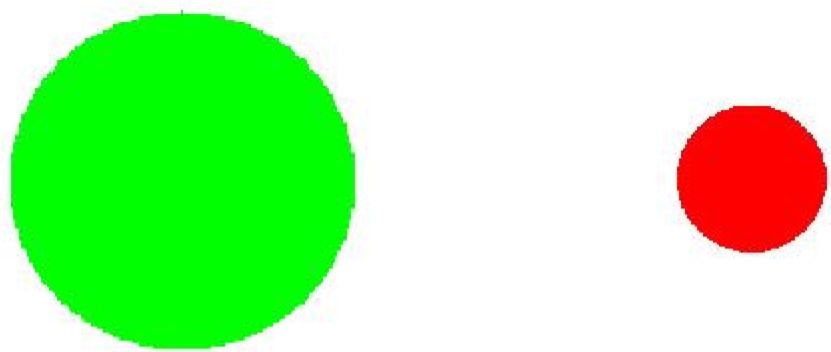
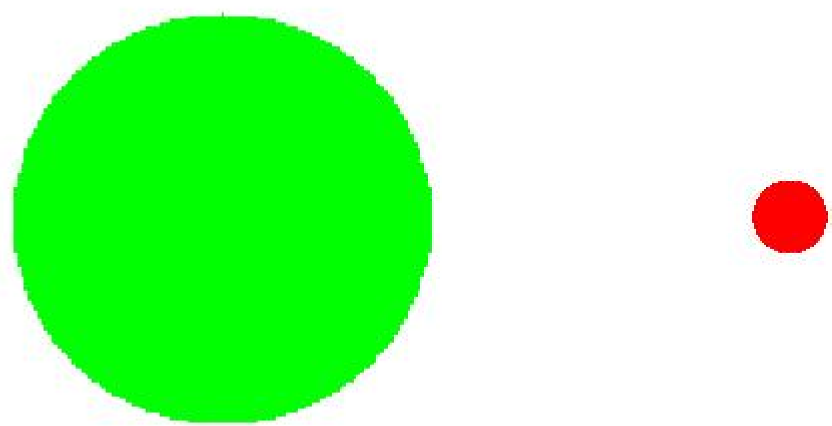
  Short answer: the volume becomes dominated by the fast-expanding regions

$$\langle\theta\rangle = 0$$

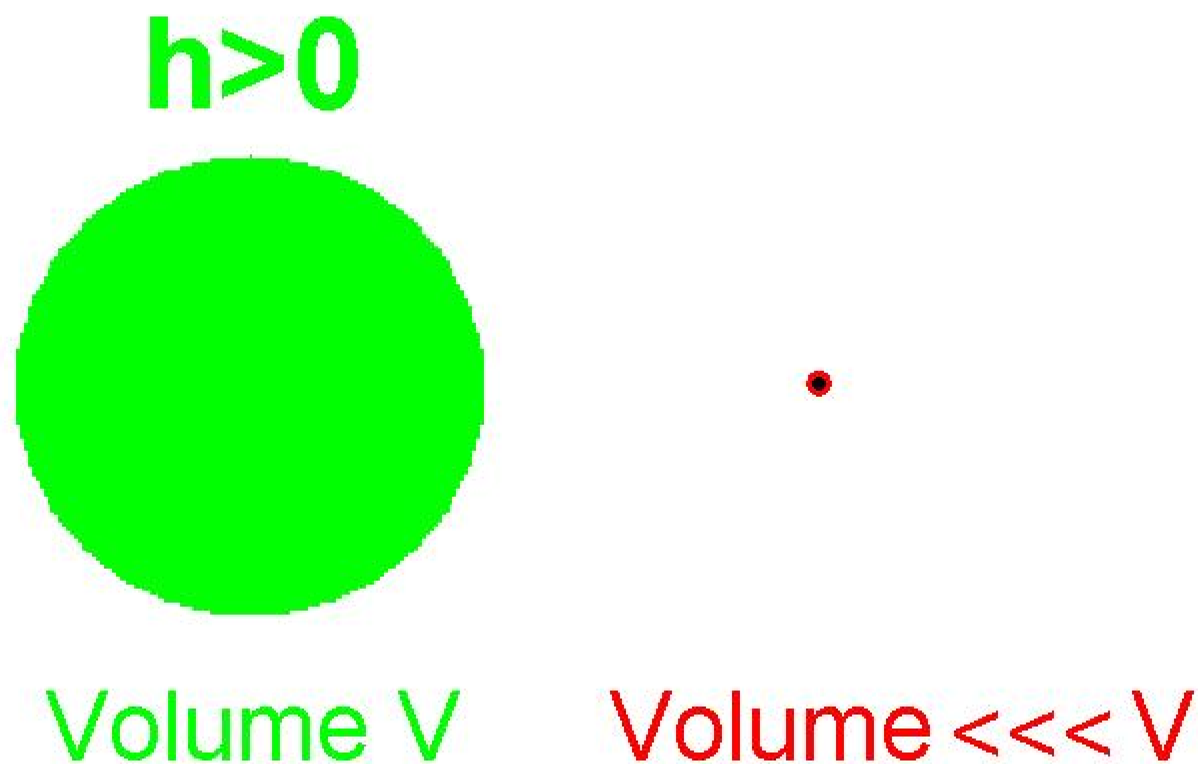
$H>0$ $-H<0$

Volume V_0 Volume V_0





$\langle\theta\rangle \simeq h > 0$



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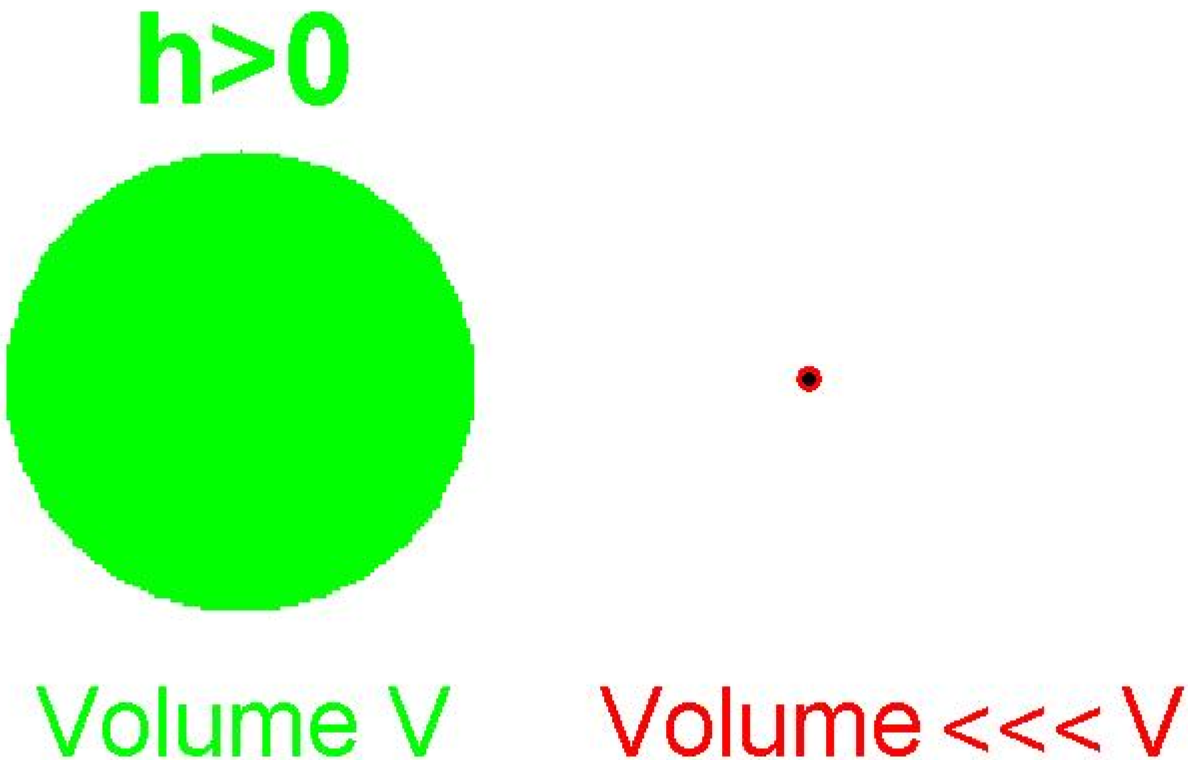

$\Longrightarrow$ expansion has accelerated from 0 to h

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  - relating the volume-averages to local observables leads to apparent acceleration

Conclusions

- Averages of inhomogeneous quantities do not evolve in time like the corresponding homogeneous quantities

- Average expansion can accelerate even if local expansion decelerates everywhere

- Structure formation may account for the observed cosmic acceleration within standard physics

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