Non-singular Warped Compactifications and the quest for a Realistic Cosmology

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Warped Compactification

- **Basic Ideas**
  - Extra Dimensions $\Rightarrow$ Higher dim spacetime: "bulk"
  - Localization of matter to a subspace "3-brane"
    (in contrast with Kaluza-Klein approach)
- **Motivations**
  - Particle Physics: Mass Hierarchy (why $\boxed{M_{EW} \ll M_{Pl}}$?)
  - Braneworld Gravity: A new Compactification Scheme

General Motivations

• **String theory with fluxes + branes: a consistent quantum theory of gravity if there are 10 or 11 dimensions in total**

• **Warping generic**

Interesting physics:

a) SUSY breaking; particle physics (model dependent)

b) Cosmology: inflation, dark energy (Universal aspect)

Specific Motivation

Observations seems to fit (nicely) with Einstein's GR with a cosmological constant term: How to get this picture from a compactified string/M- theory?

10d, 11d
String/M theory
~Unique

4d phenomenology/
Cosmology
Non-unique?

String
``Compactifications"

Warped Randall-Sundrum model

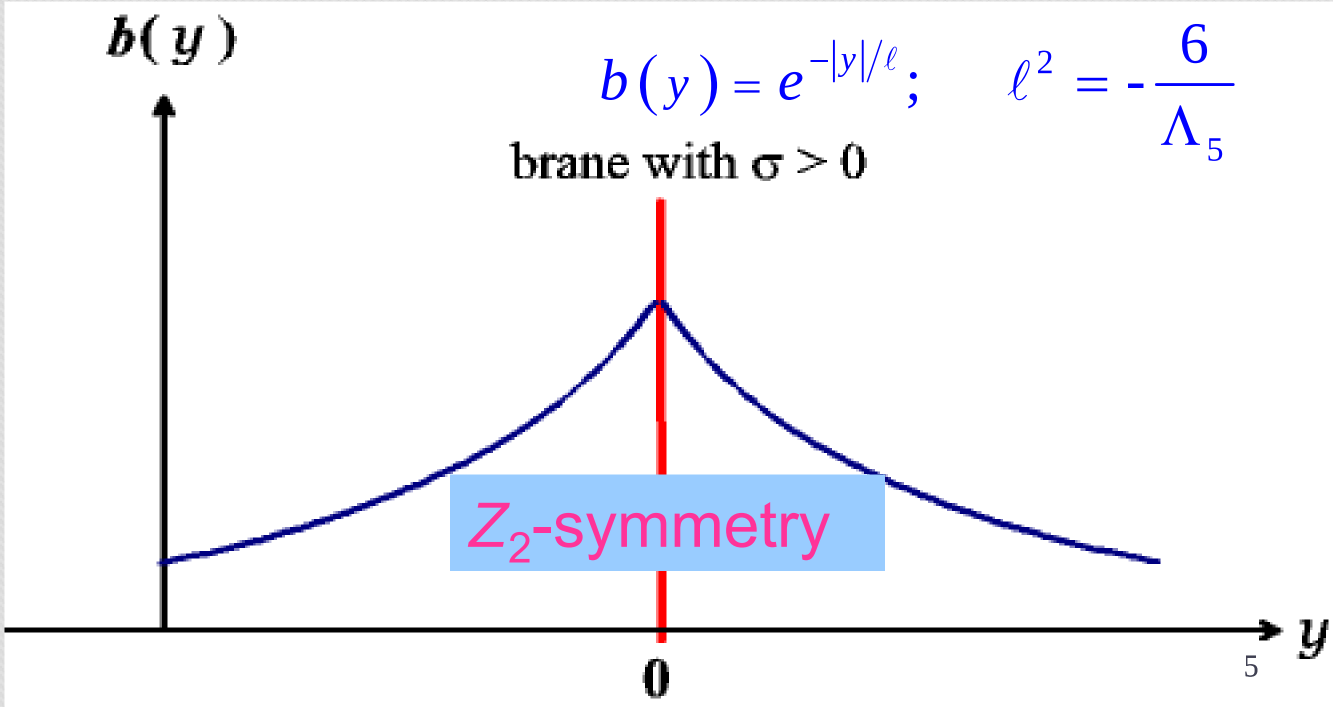
$$S = \frac{1}{2\kappa_5^2}\int_{M_5} d^5x\sqrt{-g}\left(R-2\Lambda_5\right)-\int_{\Sigma_4} d^4x\sqrt{-q}\,\sigma$$

Must be $g_{\mu\nu}$ **for a realistic cosmology**

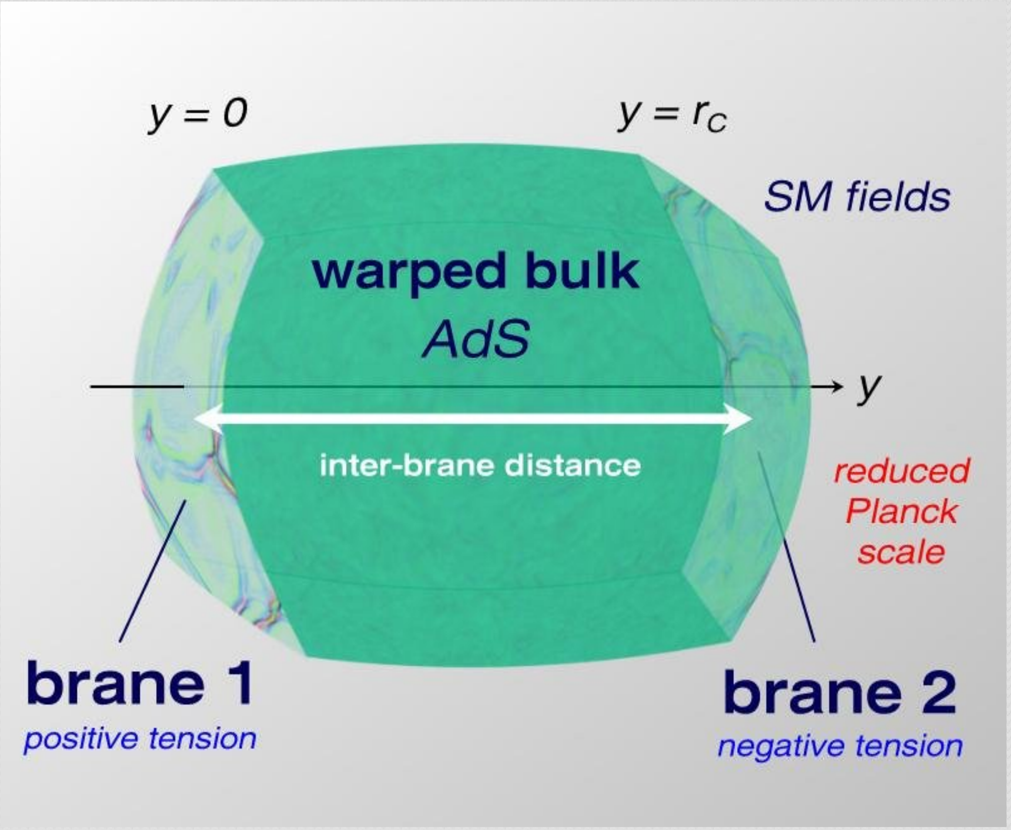
$$\Rightarrow \quad ds^2 = dy^2 + e^{-2|y|/\ell}\eta_{\mu\nu}dx^\mu dx^\nu \quad \text{for } \kappa_5^2\sigma = \sqrt{-6\Lambda_5}$$

- Warp factor $A(y)=e^{-|y|/\lambda}$ decreases exponentially as $|y|\to$infty

- Extra dim is effectively compact with size $\sim\lambda$



Two-brane Model



$$\text{Dimension of Bulk} = 1 + \text{Dimension of Brane}$$

This picture enormously changes the low energy perspective

5D de Sitter Cosmology

IPN: arxiv:0903.4190 (PLB)

- Effective Action

$$S = \frac{M_5^3}{2}\int d^5x\sqrt{-g_5}\,(R-2\Lambda_5) + \frac{M_5^3}{2}\int d^4x\sqrt{-g_b}\,(-T_3)$$

- Ansatz:

$$ds_5^2 = e^{2A(z)}\left(-dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] + \rho^2 dz^2\right)$$

- Exact solution:

$$a(t) = \frac{1}{2}e^{\mu t/\rho} + \frac{k\rho^2}{2\mu^2}e^{-\mu t/\rho},\quad A(z) = \frac{24\mu^2}{24\mu^2 e^{\mu|z|} + \Lambda_5\rho^2 e^{-\mu|z|}}$$

Dimensional reduction

$$S_{eff} \supset \frac{M_5^3 \rho}{2} \int d^4x \sqrt{-\hat{g}_4} \int dz\, \mathrm{e}^{3A(z)} \left(\hat{R}_4 - L_\Lambda - 2\Lambda_5\, e^{2A(z)} \right)$$

$$L_\Lambda = \frac{12\,\mu^2}{\rho^2} \left(1 - \frac{160\, \Lambda_5 \mu^2 \rho^2}{\left(24\,\mu^2 e^{\mu|z|} + \Lambda_5 \rho^2 e^{-\mu|z|}\right)^2} \right) - \frac{16\,\mu}{\rho^2} \left(\frac{24\,\mu^2 e^{\mu|z|} - \Lambda_5 \rho^2 e^{-\mu|z|}}{24\,\mu^2 e^{\mu|z|} + \Lambda_5 \rho^2 e^{-\mu|z|}} \right) \delta(z)$$

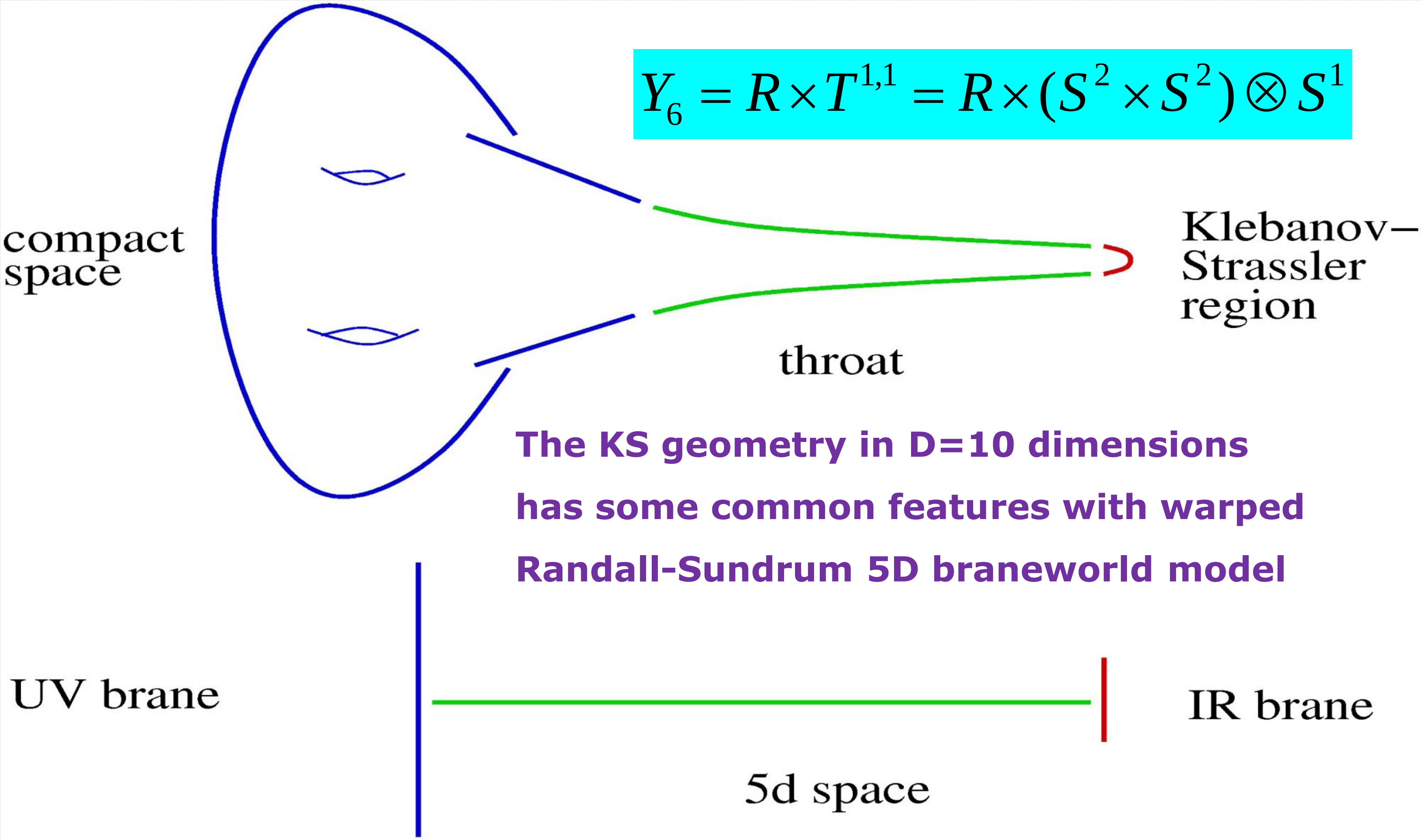
Bulk contribution

Boundary contribution

Source term / Z2 symmetry

RS type (effectively) 5d-regions can arise in warped compactifications of type IIB string theory

$$Y_6 = R \times T^{1,1} = R \times (S^2 \times S^2) \otimes S^1$$



The KS geometry in D=10 dimensions has some common features with warped Randall-Sundrum 5D braneworld model

Λ or dark energy in string theory

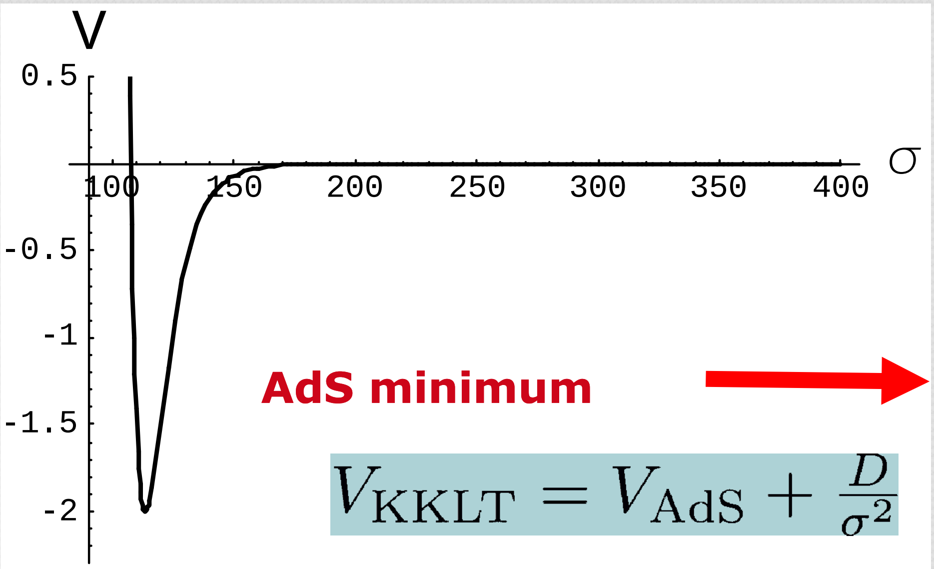
KKLT (Kachru, Kallosh, Linde, Trivedi) 2003

1) **Start with a theory with runaway potential (AdS minimum)**

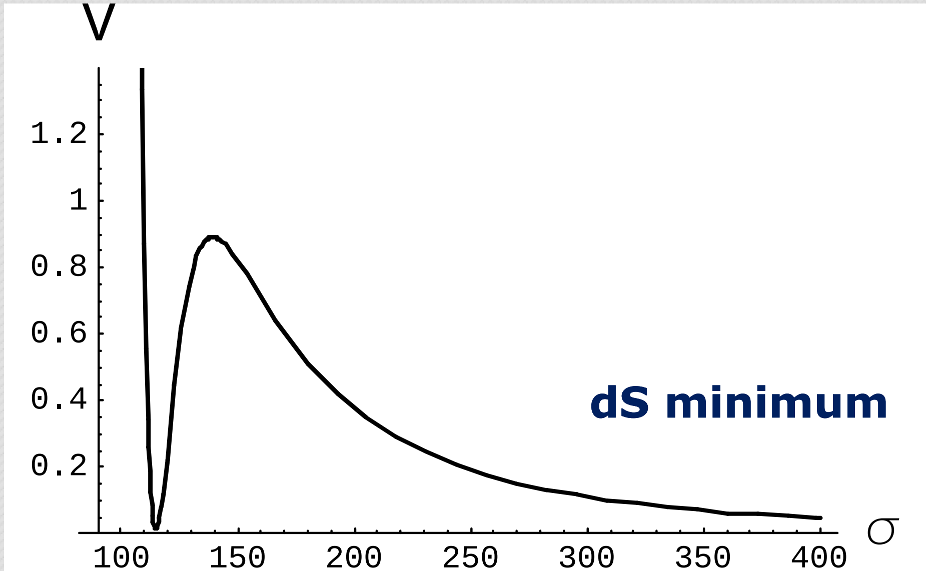
2) **Bend this potential invoking some (non-perturbative) effects**

3) **Uplift the minimum to the state with positive vacuum energy by adding a positive energy of an anti-D3 brane in warped Calabi-Yau space**

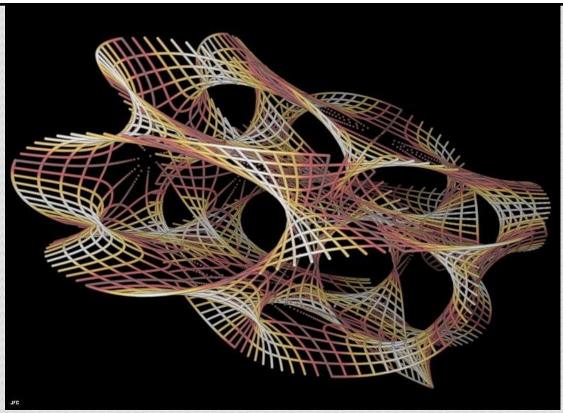
This proposal suffers from fine-tuning issues associated with the necessary flatness of the potential and or the level of fine tuning required for Λ to be the present-day gravitational vacuum (dark energy) density



$$V_{\mathrm{KKLT}} = V_{\mathrm{AdS}} + \frac{D}{\sigma^2}$$

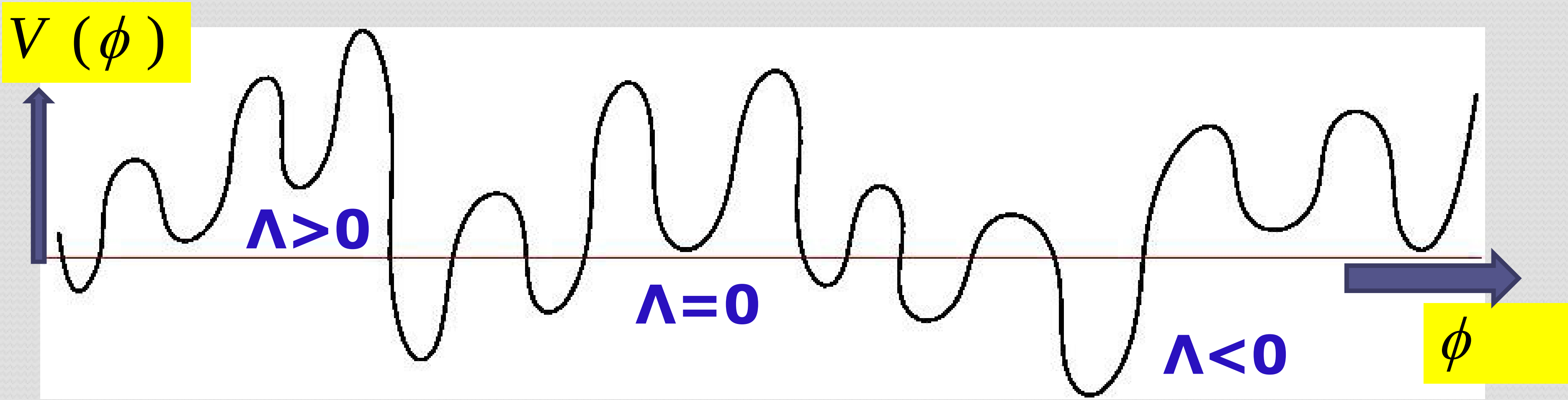


String Theory and Λ



Dark energy – likely to be linked to moduli fields, e.g., size and shape of compact space

Λ – could depend on fundamental (UV) structure of theory and statistics of string vacua: internal space geometry, branes, fluxes, etc



What are the obstacles for finding de Sitter solutions in string/M theory?

A celebrated answer

Brane World 'no-go theorem'

'No-go theorem' forbids cosmic acceleration in cosmological solution arising from compactification of pure SUGRA where the internal space is time-dependent, non-singular compact manifold without boundary.

- Gibbon (1984)
- Maldacena-Nunez (2001)

Why?

Acceleration requires violation of 4D strong energy condition

$$R_{00}^{(4)} < 0 \ \text{ or } \ T_{AB}\xi^A \xi^B < 0$$

If extra dimensions are warped and static, then in a compactified theory

$$\boxed{R_{00}^{(4)} \geq 0}$$

Provided that SEC holds for D=10 or 11D SUGRA

$$\boxed{R_{00}^{(D)} \geq 0}$$

Why should the SEC be violated?

To see this one considers a FRW metric

$$ds_4^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right)$$

The time-time component of *4D* Ricci tensor gives

$$R_{00}^{(4)} = -\frac{3\ddot{a}}{a}$$

Acceleration requires $\dot{a}/a > 0$ **and** $\ddot{a}/a > 0$ **hence**

$$R_{00}^{(4)} < 0$$

The 4D Einstein field equations imply that

$$R_{00}^{(4)} = T_{00} + g^{ij}T_{ij} = \rho + 3p$$

An accelerated expansion is possible in a universe governed by Einstein's gravity only if the matter in it violates SEC.

A celebrated version of "no-go" theoreom

Consider a (4+m)-dimesional metric ansatz

$$\mathrm{ds}_{\mathrm{D}}^{2} = e^{2A(y)} ds_4^2(x) + g_{mn}(y) dy^m dy^n$$

This gives

$$\mathrm{R}_{00}^{(\mathrm{D})}(x,y) = R_{00}^{(4)} - \frac{1}{4} e^{-2A(y)} \nabla_y^2 e^{4A(y)}$$

$$\Rightarrow \left[e^{2A(y)}\right] R_{00}^{(4)} = \int e^{2A(y)} R_{00}^{(D)} + \frac{1}{4} \int \nabla^2 e^{4A(y)}$$

If the last term above vanishes, then

$$\mathrm{R}_{00}^{(D)} \geq 0 \;\; \text{only if} \;\; \mathrm{R}_{00}^{(4)} \geq 0$$

Any time that one does not understand something, one can point to details that do not work.

It is important to identity what is wrong qualitatively and give the best clues to future progress.

1. **Allow internal space to be time-dependent, analogue of time-dependent scalar fields – Lukas et al '00**

2. **Drop condition that internal space is flat or positive, it may be negatively curved (hyperbolic)**

Townsend-Wohlfarth, N. Ohta, IPN et al 2003

A compactified theory on hyperbolic spaces leads to cosmologies with transient accelerating phase.

SUGRA solutions describing accelerating cosmologies from twisted spaces:

IPN&Wiltshire: hep-th/0502003 (PLB), hep-th/0504135 (PRD), IPN: hep-th/0609086 (PRL)

Recent Progress

The limitation with warped models studied previously have arisen from an over simplification of 10d metric ansatz.

IPN: arXiv:0901.2568 [hep-th]

IPN: arXiv:0903.4190 [hep-th] (PLB)

IPN: Non-singular warped compatification, to appear

These papers give a few explicit examples of non-singular warped compactification on de Sitter spaces.

An explicit example of de Sitter compactification

$$ds_{10}^2 = e^{2A(y)} ds_4^2(x) + e^{-\alpha A(y)} ds_6^2(y)$$

$$ds_4^2(x) = -dt^2 + a(t)^2 \, dx_3^2$$

$$\begin{aligned} ds_6^2 &\equiv g_{mn}(y) dy^m dy^n \\ &= dy^2 + \alpha_1 y^2 ds_{X_5}^2 \end{aligned}$$

S^5 or $T^{1,1}$

$$R_6 = \frac{20(1-\alpha_1)}{\alpha_1 y^2}$$

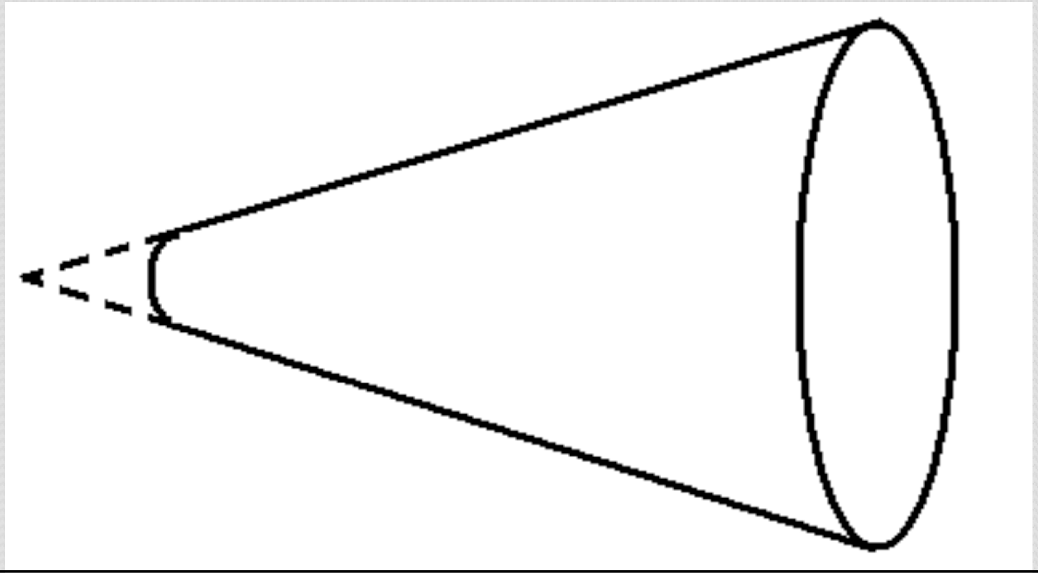
A simple example of Y6 is the non-compact Calabi-Yau

The base of the conifold is Einstein-Sasaki space

$$T^{1,1} = (SU(2) \times SU(2)) / U(1)$$

$$g_{ij} d\theta^i d\theta^j = \frac{1}{9}(d\psi + \cos\theta_1 d\varphi_1 + \cos\theta_2 d\varphi_2)^2 + \frac{1}{6}\sum_{a=1}^{2}(d\theta_a^2 + \sin\theta_a^2 d\varphi_a^2)$$

y=0



The Ansatz

$$ds_{10}^2 = e^{2A(y)} ds_4^2(x) + e^{-\alpha A(y)} ds_6^2(y)$$

$$ds_6^2 \equiv g_{mn}(y) dy^m dy^n = \rho^2 (dy^2 + \alpha_1 y^2 ds_{X_5}^2)$$

solves the 10d vacuum Einstein equations when

$$e^{(\alpha+2)A} = \frac{3(\alpha+2)^2}{32} \frac{y^2}{L^2} \qquad \alpha_1 = \frac{(\alpha+2)^2}{8}$$

$$a(t) \propto e^{Ht} \qquad H = \sqrt{\frac{1}{\rho^2 L^2}}$$

A drawback of this solution is that the 6d metric and hence the warp factor is singular at y=0

Why did the previous authors -- including Gibbons, Maldacena-Nunez, Giddings et al and many others -- not realise (or somehow rule out) the above explicit de Sitter solutions of 10D Einstein equations?

Warping generic:

$$ds_{10}^2 = e^{2A(y)} d\hat{s}_4^2(x) + e^{-2A(y)} g_{mn} dy^m dy^n$$

$$G_\mu^{\ \nu} = \kappa_{10}^2 T_\mu^{\ \nu}, \qquad G_m^{\ n} = \kappa_{10}^2 T_m^{\ n} \Rightarrow$$

$$\nabla_y^2 A = \frac{1}{4} e^{-4A} \hat{R}_4 + \frac{\kappa_{10}^2}{8} e^{-2A} \left(T_m^{\ m} - T_\mu^{\ \mu} \right)$$

= 0 flat space
> 0 de Sitter space

> 0
with p-brane, p<7
with q-flux, q>1

with $\oint \nabla_y^2 A = 0$ **de Sitter solutions are not possible**

Giddings, Kachru & Polchinski 2001

Warping generic, but NOT the proof

$$ds_{10}^2 = e^{2A(y)} d\hat{s}_4^2(x) + e^{-\alpha A(y)} g_{mn} dy^m dy^n$$

$$G_\mu^{\ \nu} = \kappa_{10}^2 T_\mu^{\ \nu}, \qquad G_m^{\ n} = \kappa_{10}^2 T_m^{\ n} \Rightarrow$$

$$\nabla_y^2 A = \frac{1}{4} e^{-(2+\alpha)A} \hat{R}_4 - 2(2-\alpha)(\partial_y A)^2 + \frac{\kappa_{10}^2}{8} e^{-\alpha A}\left(T_m^{\ m} - T_\mu^{\ \mu}\right)$$

$$\nabla_y^2 e^{(2+\alpha)A} = \frac{2+\alpha}{4} \hat{R}_4 - \frac{2-3\alpha}{2+\alpha} e^{-(2+\alpha)A} (\partial_y A)^2 + \frac{2+\alpha}{8} \kappa_{10}^2 \left(T_m^{\ m} - T_\mu^{\ \mu}\right)$$

with $\oint \nabla_y^2 e^{(2+\alpha)A} = 0$ **de Sitter solutions are possible**

IPN: arxiv:0901.2568

10D Supergravity Action

$$I_{10-D} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{10}}\Big[R_{10} - \tfrac{1}{2}(\partial\Phi)^2 - \tfrac{1}{12}e^{-\Phi}(\partial B_2)^2 - \tfrac{1}{2}e^{2\Phi}(\partial C)^2$$

$$- \tfrac{1}{12}e^{\Phi}(\partial C_2 - C\partial B_2)^2 - \tfrac{1}{4\times 5!}F_5^2\Big] - \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\varepsilon_{10}C_4\partial C_2\partial B_2$$

$$(\partial B_2)_{...} = 3\partial_{[.}B_{..]},\ (\partial C_4)_{.....} = 5\partial_{[.}C_{....]},\ \mathrm{F}_5 = \partial C_4 + 5(B_2\partial C_2 - C_2\partial B_2)$$

$$\nabla_y^2 A = \frac{1}{4} e^{-(2+\alpha)A} \hat{R}_4 - 2(2-\alpha)(\partial_y A)^2$$

$$+ \frac{\kappa_{10}^2}{8} e^{-\alpha A} \left(T_m^{\ m} - T_\mu^{\ \mu} \right)$$

$$+ \text{fluxes}$$

Fluxes generally contribute positively

The two major assumptions that went into the earlier discussions of braneworld no-go theorems are

1.

$$V_6^{\mathrm{w}} = \int d^6 y \sqrt{g_6}\, e^{(2-3\alpha)A} = const\ ,$$

$$\frac{1}{G_N^{\mathrm{eff}}} \equiv \frac{M_{10}^8 \times V_6^{\mathrm{w}}}{(2\pi)^6}$$

The natural "constants" change neither in time nor with space when one moves away from the 4d hypersurface!

2.

$$\int \nabla_y^2\, e^{n\,A(y)} = 0$$

(with arbitrary n)

These constraints are 'strict' which are generally *not* satisfied by cosmological solutions, especially, in the presence of some localised sources like branes

An explicit example: non-singular solution

$$ds_{10}^2 = e^{2A(z)}\left(-dt^2 + a(t)^2 d\vec{x}_3^2\right) + e^{-\alpha A(z)} ds_6^2(z)$$

$$ds_6^2 \equiv g_{mn}(z) dz^m dz^n$$

$$= \rho^2 (\sinh^2(z + z_0) + \alpha_1 \cosh^2(z + z_0) ds_{X_5}^2)$$

This solves the 10D Einstein equations when

$$A(y) = \frac{2}{2+\alpha} \ln\left(\cosh(z + z_0) \times \sqrt{\frac{3(2+\alpha)^2 \mu^2}{32}}\right)$$

$$\alpha_1 = \frac{(2+\alpha)^2}{8}$$

$$a(t) \propto e^{Ht}$$

$$H = \sqrt{\frac{\mu^2}{\rho^2}}$$

$$ {}^{(10)}R_{00}(x,y) = {}^{(4)}\hat{R}_{00}(x) + {}^{(4)}\hat{R}_{00}(x) + \frac{\mu^2}{\rho^2}\left(3 + \frac{3(2+\alpha)}{8}\coth(z+z_0)\delta(z)\right) $$

It is possible to get acceleration $\hat{R}_{00} < 0$ **without violating the 10D strong energy condition** ${}^{(10)}R_{00} \geq 0$

$$ ds_{10}^2 = e^{-A_0} \left(\frac{3(2+\alpha)^2 \mu^2 \cosh^2 z}{32}\right)^{2/(2+\alpha)} \times \left(ds_4^2 + \frac{32\rho^2}{3(2+\alpha)^2\mu^2}\tanh^2 z \left(dz^2 + \frac{(2+\alpha)^2}{8}\coth^2 z\, ds_{X_5}^2 \right)\right) $$

$$ V_6 \sim (\rho/\mu)^6 \ln\cosh z \sim z $$

Dimensional Reduction

$$\frac{6\mu^2}{\rho^2}$$

$$ds^2_{10} \sim M^8_{10}\int d^{10}x\sqrt{-g_{10}}\left(R_{(10)}+\ldots\ldots\right)$$

$$=\frac{e^{-4A_0}M^8_{10}\times V_6}{\rho^6}\int d^4x\sqrt{-\hat{g}_4}\,(\hat{R}_{(4)}-2\Lambda_4)$$

$$-\frac{e^{-4A_0}M^8_{10}V_5}{\rho^6}\int\left(\cosh z\right)^{16/(2+\alpha)}\delta(z)\,dz$$

If one wants to tune $\Lambda_4 \sim 10^{-120}M^2_{Pl}$ then one is required to take $A_0 \geq 136$ provided $M_{10} \geq TeV$

It is NOT natural to just set Λ_4 to zero!

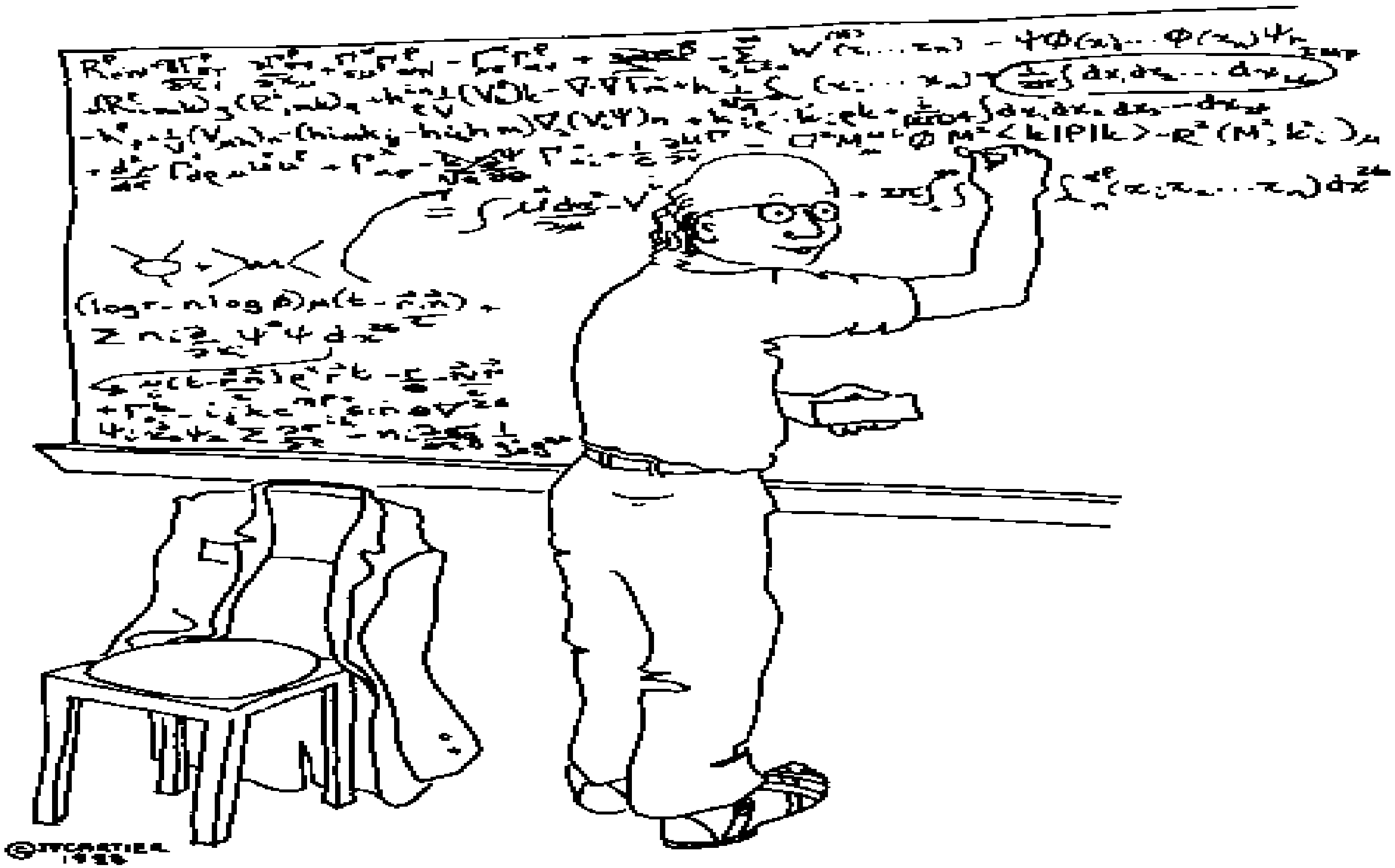
Summary

Warped compactifications:

- **Possibly ubiquitous among geometric solutions and provide new pictures of the universe and also new opportunities for studying cosmology.**

- **Potentially quite rich phenomenology: dS, hierarchy, inflation,….**

- **Cosmic acceleration (attributed vacuum energy or Λ) is an extra-dimensional phenomenon.**

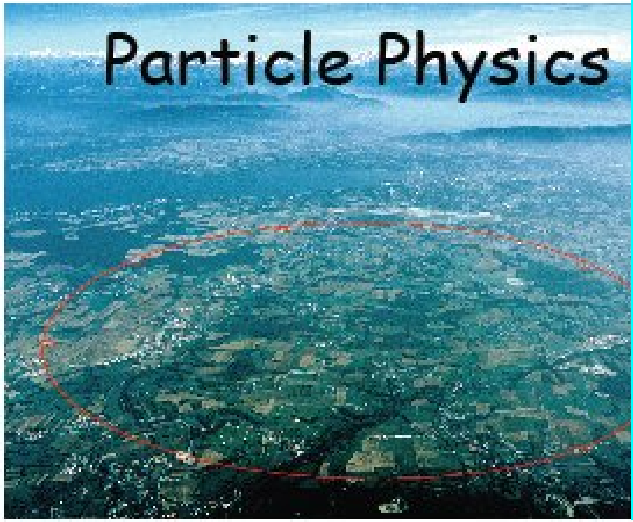


"At this point we notice that this equation is beautifully simplified if we assume that space–time has 92 dimensions."

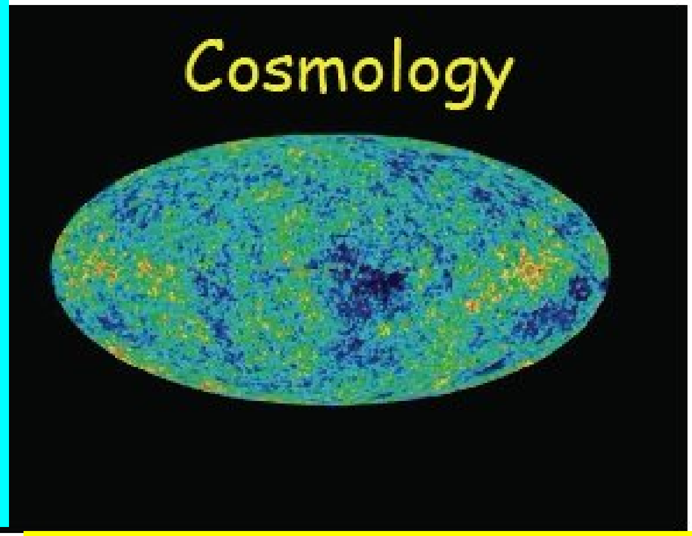
Thanks

Warped compactifications

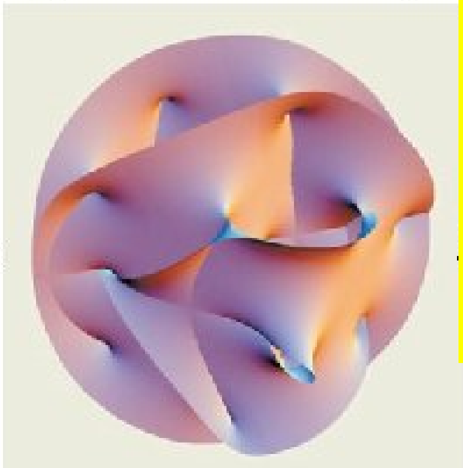
Summary



Cosmic acceleration (inflation or dark energy) is an extra-dimensional Phenomenon in many higher dimensional gravity models



Potentially quite rich phenomenology: dS, hierarchy, inflation,....



Ubiquitous among geometric solutions and provide new pictures of the universe and also new opportunities for studying cosmology.

Cosmology + LHC !