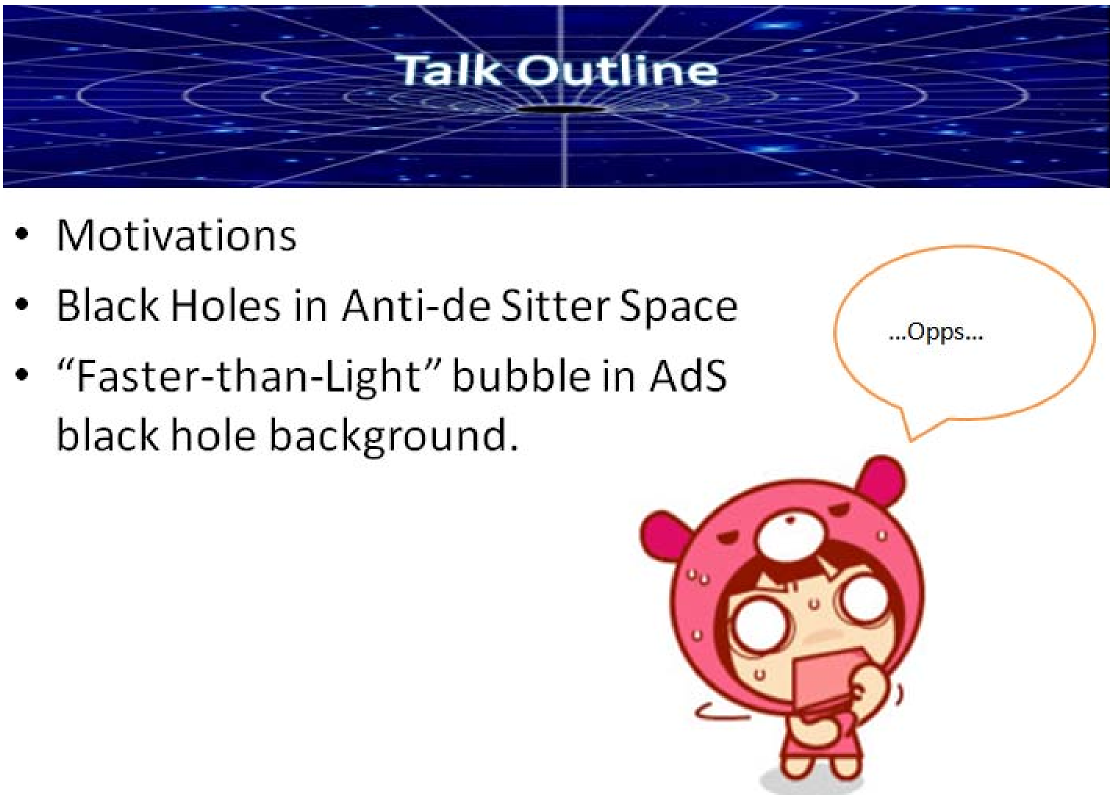
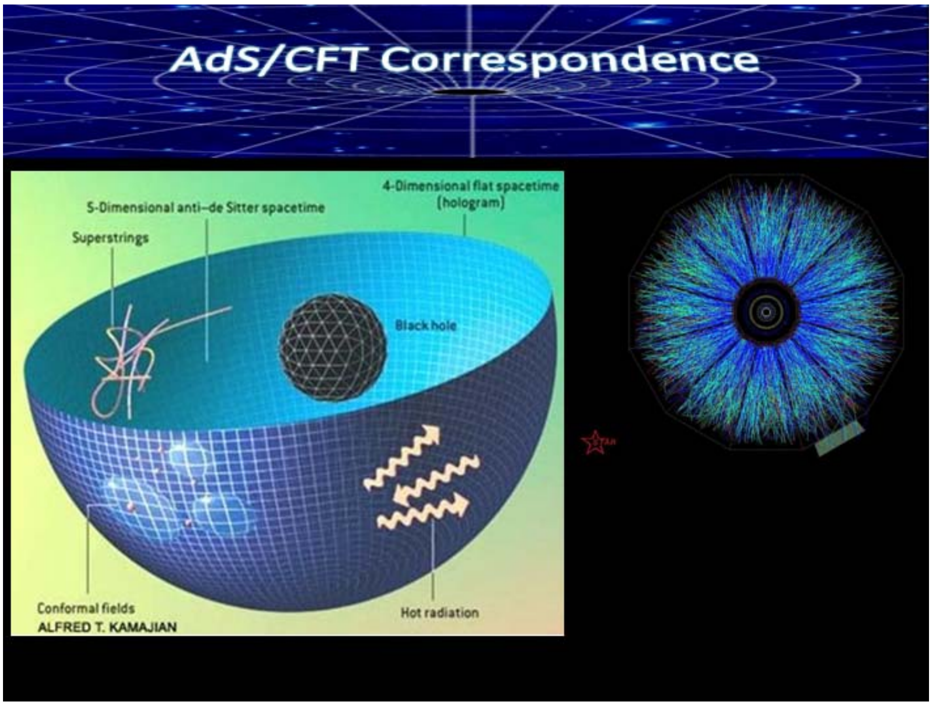
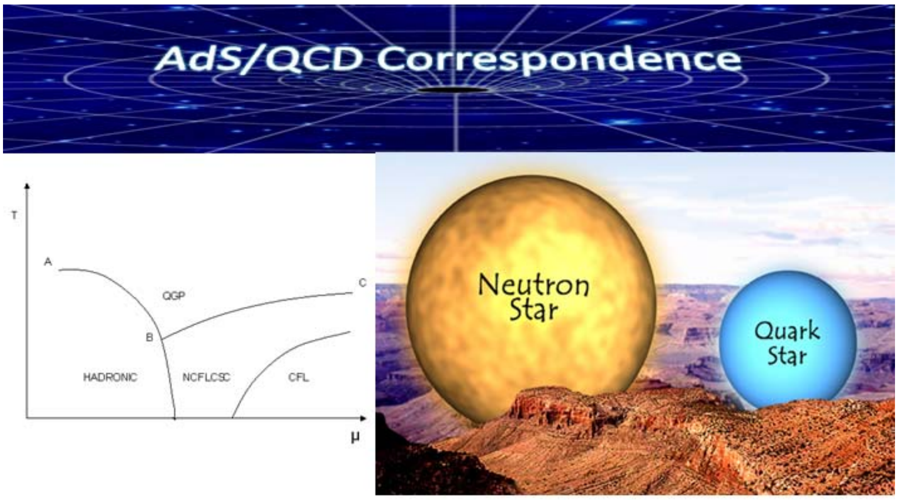
As the first slide suggested, my original plan was to talk about properties of FTL (Faster-than-Light) drive, also known as "warp drive" especially among science fiction fans.



However, as I was preparing for the talk, I realized I have deviated from the topic… and so my talk is now concentrated on black holes in Anti-de Sitter space, which will be largely introductory, though there is still a mention on FTL drive as a side comment.

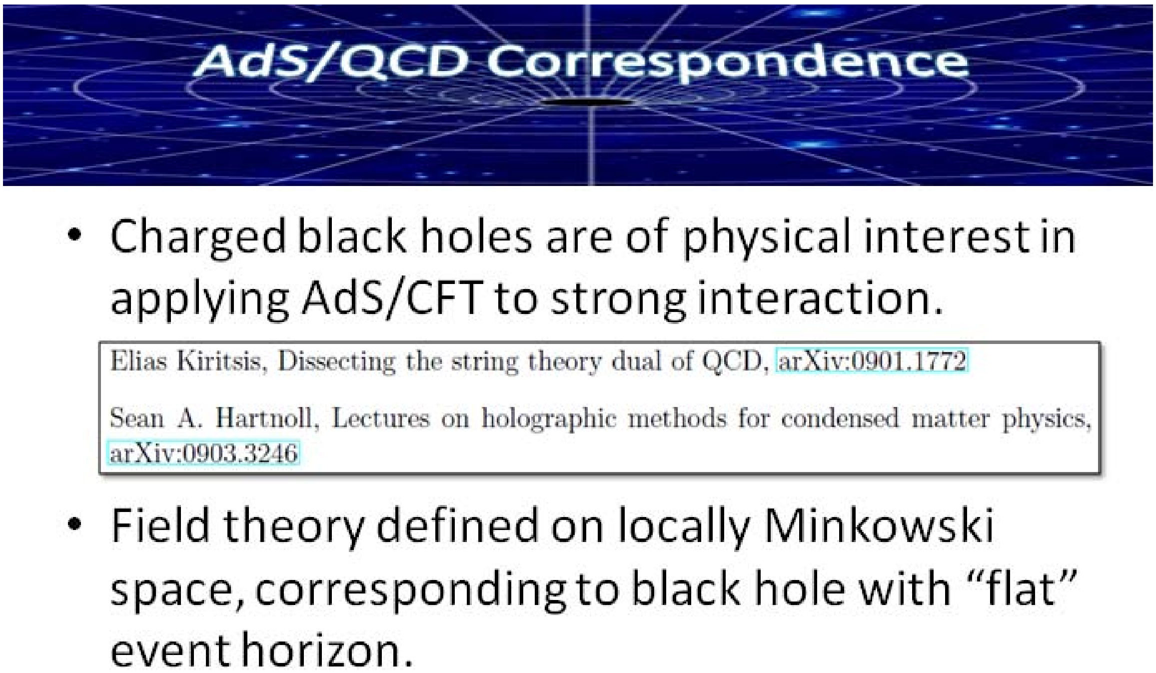


The motivation behind the study of black holes in Anti-de Sitter space is mainly in view of AdS/CFT correspondence, which we recall, gives conformal field theory on the boundary a dual description in terms of gravitational theory in the AdS bulk. In particular, one may want to understand the strong interaction (say, the quark gluon plasma state, where quarks are de-confined) via its gravitational side of the story. This is referred to as AdS/QCD correspondence.

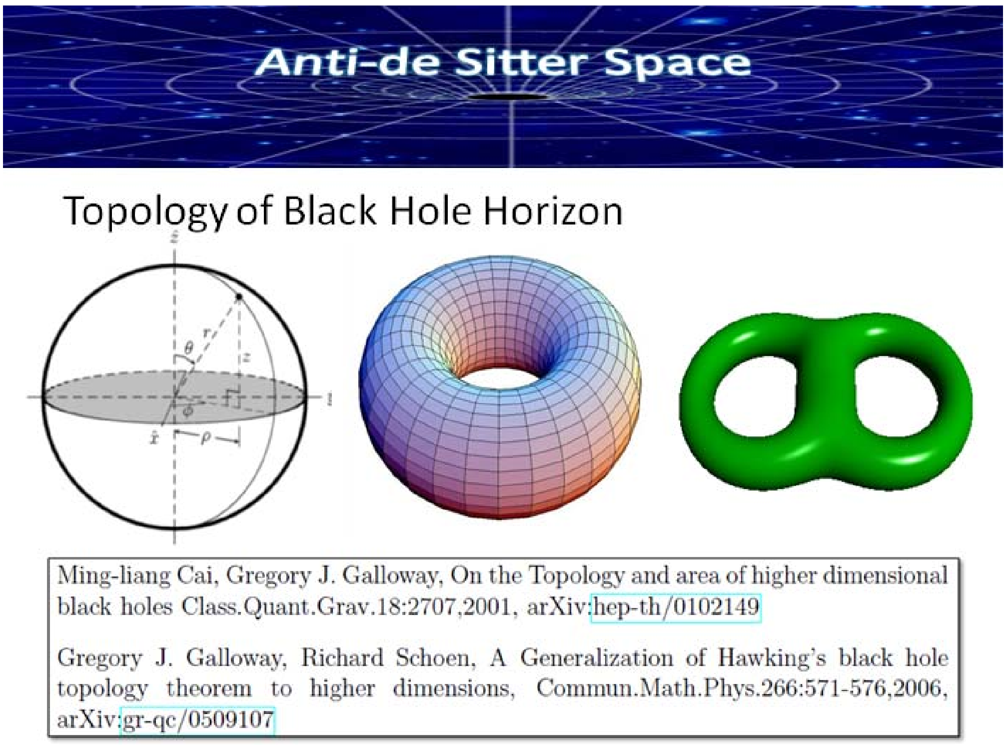


Brett McInnes, Holography of the Quark Matter Triple Point, arXiv:0910.4456

Interestingly, it is possible that in the dense core of neutron stars, matter exists in the QGP phase. Neutron stars (or maybe quark stars, if they exist) are the closest thing we have prior to total gravitational collapse into black holes. But it seems that, if the AdS/QCD approach is trustable, then we can make use of gravity system, in particular black holes in 5-dimension, to understand the state of matter at the core of neutron stars in *our 4-dimensional universe*.



In spacetime satisfying the dominant energy condition, there is strong constraint on the allowed topology for the black hole horizon: in particular, for black holes in asymptotically flat and asymptotically de Sitter spacetime, the horizons must be topologically a sphere. Is there a spacetime which allow black hole to have non-trivial topology, say, a torus, or other shapes with higher genus?

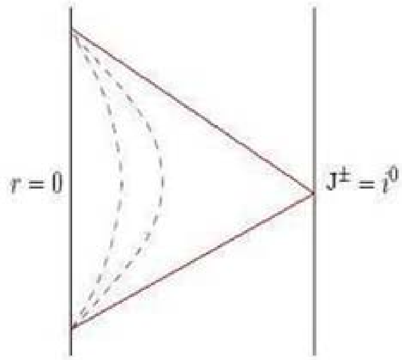


It turns out that if we put a black hole in Anti-de Sitter space, then it is possible for its horizon to be topologically any compact constant curvature manifold, including flat and negatively curved. This is not surprising as AdS space does not satisfy the dominant energy condition.

Anti-de Sitter Space

$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \left(1+\frac{r^2}{L^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

- AdS does not satisfy Dominant Energy Condition, and not globally hyperbolic.



A d-dimensional anti-de Sitter space, denoted by AdS_d, is a maximally symmetric spacetime with constant negative curvature. Unlike the de Sitter case, AdS space corresponds to solution of Einstein Field Equations with *negative cosmological constant*, i.e. $\Lambda = -\frac{(d-2)(d-1)}{2l^2} < 0$.

Black Holes in Anti-de Sitter Space

The generalized Kottler metric in d-dimensional spacetime with coordinate labelled by $x^\mu = (t, r, x^i)$ where $1 \leq i \leq d-2$ is given by

$$\boxed{ds^2 = -f(r)dt^2 + [f(r)]^{-1}dr^2 + r^2 h_{ij}(x)dx^i dx^j}$$

where $f(r) = \left(k - \frac{\omega_d m}{r^{d-3}} \pm \frac{r^2}{l^2}\right)$, for which $k = -1, 0, +1$, and $\omega_d = \frac{16\pi G}{(d-2)\mathrm{Vol}(M^{d-2})}$, $\mathrm{Vol}(M^{d-2}) = \int d^{d-2}x\sqrt{h}$; while l is a length scale related to the cosmological constant $\Lambda := \mp\frac{(d-1)(d-2)}{2l^2}$.

Danny Birmingham, Topological Black Holes in Anti-de Sitter Space, Class.Quant.Grav. 16 (1999) 1197, arXiv:hep-th/9808032

Black Holes in Anti-de Sitter Space

Consider a non-charged black hole in AdS:

$$\mathrm{g}(\mathrm{AdSSch_k}) = -\left[\frac{\mathrm{r}^2}{\mathrm{L}^2} + \mathrm{k} - \frac{16\pi\mathrm{M}}{3\Gamma_\mathrm{k}\mathrm{r}^2}\right]\mathrm{dt}^2 + \frac{\mathrm{dr}^2}{\frac{\mathrm{r}^2}{\mathrm{L}^2} + \mathrm{k} - \frac{16\pi\mathrm{M}}{3\Gamma_\mathrm{k}\mathrm{r}^2}} + \mathrm{r}^2\mathrm{d}\Omega_\mathrm{k}^2.$$

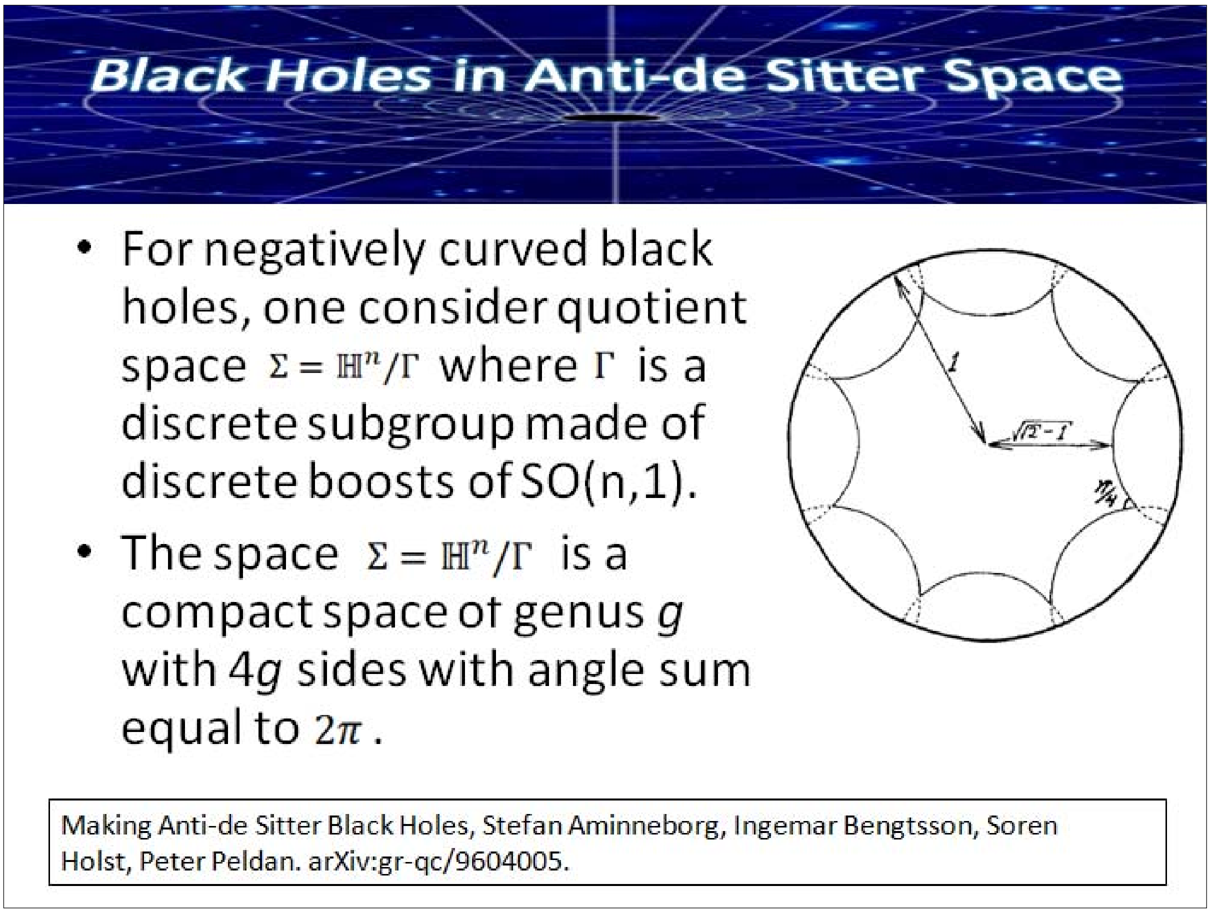
Γ_k is not uniquely defined: For each k there are many spaces of constant curvature k.

In the previous slide, we consider a "Schwarzschild" black hole in (4+1)-dimensional Anti-de Sitter space. We allow the black hole to have either negatively curved, flat, or positively curved horizon. Γ_k denotes the unit compact manifold that serves as the horizon.

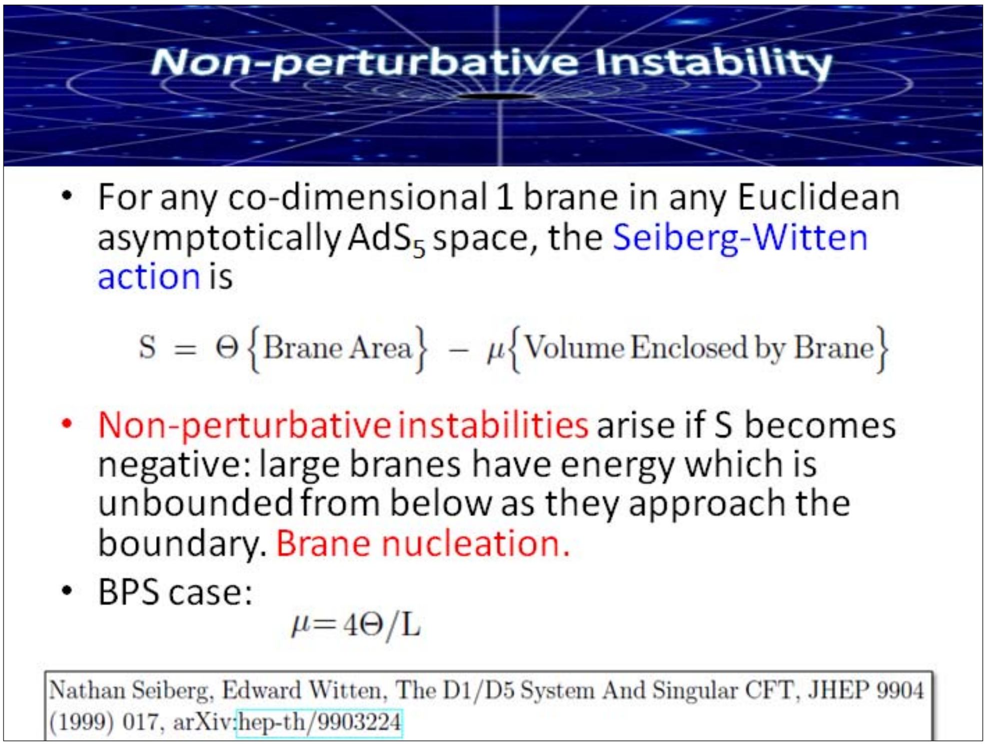
Black Holes in Anti-de Sitter Space

$k = 1$	$k = 0$
Γ_1 is fixed by topology	6 possible topologies in orientable case.* Various continuous parameters.
E.g. $S^3 \Rightarrow \Gamma_1 = 2\pi^2$ $\mathbb{R}P^3 \Rightarrow \Gamma_1 = \pi^2$	Torocosm T^3, Dicosm $T^3/\mathbb{Z}_2$, Tricosm $T^3/\mathbb{Z}_3$, Tetracosm $T^3/\mathbb{Z}_4$, Hexacosm $T^3/\mathbb{Z}_6$, Didicosm (Hantzsche-Wendt Space) $T^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$
Infinitely many 3-manifolds of constant positive curvature: $S^3/\mathbb{Z}_n$	

- *Theorem 3.5.5, J.A. Wolf, *Spaces of Constant Curvature*
- John Horton Conway, Juan Pablo Rossetti, *Describing the Platycosms*, arXiv:math/0311476



Not all black holes in AdS are stable. In particular, negatively curved black holes are non-perturbatively unstable in String theory – the Seiberg-Witten action must be positive for the black hole to be stable. Note that the action is defined for the Euclidean version of the metric.



Brane Geometry

$$\text{Area} = \sqrt{g_{tt}} \int_0^{PL} dt \int r^3 d\Omega_k = PLA_k \left[\frac{r^2}{L^2} + k - \frac{16\pi M}{3A_k r^2}\right]^{\frac{1}{2}}$$

$$\text{Volume} = \int_0^{PL} dt \int_{r_{eh}}^{r} r^3 dr' \int d\Omega_k = PLA_k \left[\frac{r^4 - r_{eh}^4}{4}\right]$$

Seiberg-Witten Action of AdS Black Hole

$$S = \Theta PLA_k \left\{ r^3 \left[\frac{r^2}{L^2} + k - \frac{16\pi M}{3A_k r^2} \right]^{1/2} - \frac{r^4 - r_{eh}^4}{L} \right\}$$

$$= \Theta PLA_k \left\{ \frac{L[kr^2 - \frac{16\pi M}{3A_k}]}{1 + \left[1 + \frac{kL^2}{r^2} - \frac{16\pi ML^2}{3A_k r^4}\right]^{1/2}} + \frac{r_{eh}^4}{L} \right\}.$$

It can be shown that for positively and negatively curved horizons, namely, with k=1 or k=0, the action is positive. Thus these black holes are non-perturbatively stable. In the case of flat event horizon, the action does not grow as it does in positively curved case, and as such the stability may not be as secure when we introduce matter fields etc. and need to be taken with cares depending on the particular case.

In application to AdS/QCD, one would ultimately want to study charged black holes in AdS, see [arXiv:hep-th/0905.1180: Bounding the Temperatures of Black Holes Dual to Strongly Coupled Field Theories on Flat Spacetime, Brett McInnes].

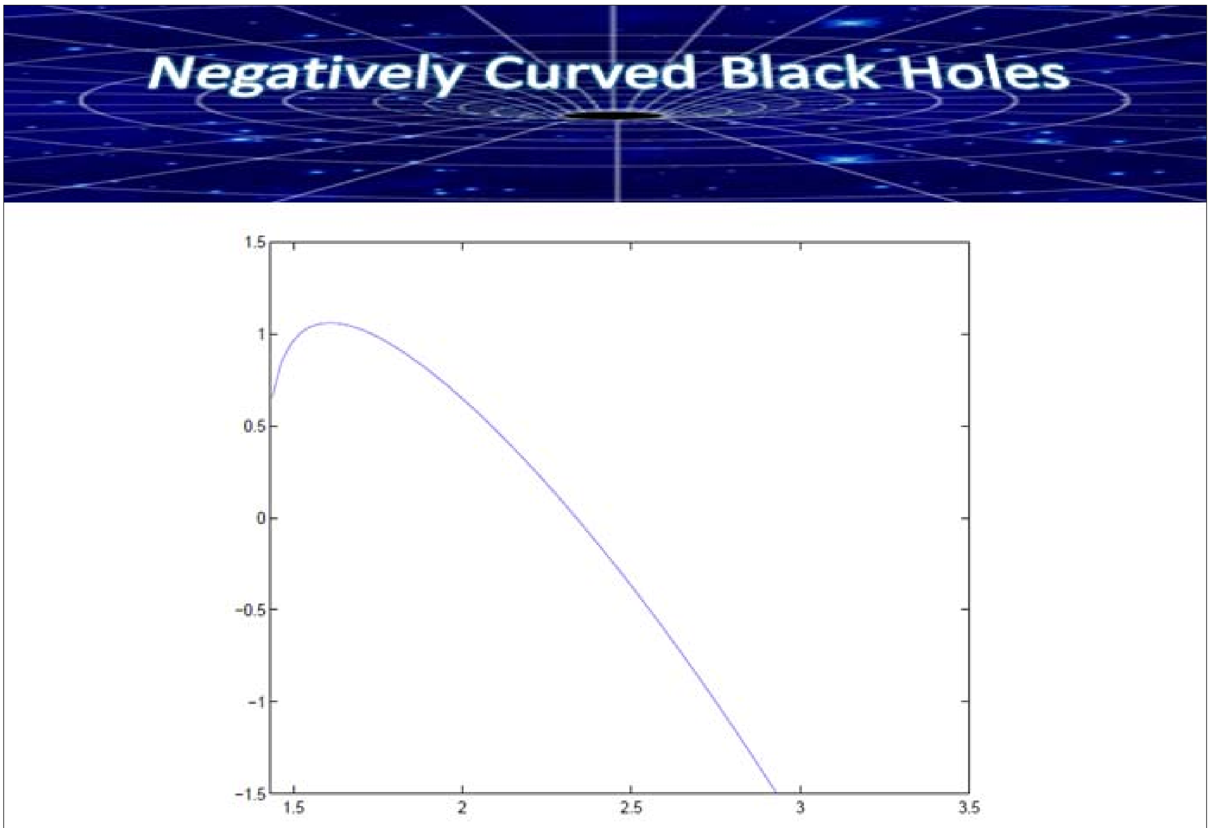


Figure : Brane Action, Negatively Curved Event Horizon.

Unlike the positively curved and flat cases, the negatively curved black holes in AdS are not stable due to the Seiberg-Witten action being unbounded from below, which leads to instability of the geometry with brane nucleation at the boundary.

Naively one may try to make the action positive for negatively curved black hole by making the volume integral negative, say by including a warp drive like effect $dr \to dr - v_s f(r) dt$, at which point we recall the basic facts about warp drive: negative energy is required to maintain the warp bubble even at sub-light speed [arXiv:gr-qc/0406083: Fundamental Limitations on Warp Drive Spacetimes, Francisco S. N. Lobo, Matt Visser]. The warp drive also exhibits closed timelike curves (causality violating). [arXiv:gr-qc/9907026].

Quick Review of Alcubierre Drive

Class. Quantum Grav. 11-5, L73-L77 (1994).

$$ds^2 = -dt^2 + \left(dx - v_s f(r_s)\, dt\right)^2 + dy^2 + dz^2 \; .$$

$$v_s(t) = \frac{dx_s(t)}{dt} \; , \qquad r_s(t) = \left[\left(x - x_s(t)\right)^2 + y^2 + z^2\right]^{1/2} \; ,$$

$$f(r_s) = \frac{\tanh\left(\sigma\,(r_s + R)\right) - \tanh\left(\sigma\,(r_s - R)\right)}{2\tanh(\sigma R)} \; ,$$

$$\lim_{\sigma \to \infty} f(r_s) = \begin{cases} 1 & \text{for} \quad r_s \in [-R, R] \; , \\ 0 & \text{otherwise} \; . \end{cases}$$

However to ensure stability in the Seiberg-Witten sense we must have the action to be positive for *all* branes. So even though for *some* branes the action may be positive when we probe the geometry of the negatively curved black holes, that would not in anyway help in establishing stability.