Equatorial circular orbits in the Kerr–Newman–de Sitter spacetimes

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Introduction

- We present influence of the cosmological constant on the character of the geodetical equatorial circular orbits in the rotating and charged Kerr-Newman black-hole and naked-singularity spacetimes.
- This work is a follow-up of previous papers [Stuchlik and Hledik 2000] and [Stuchlik and Slany 2004] .

Kerr–Newman–de Sitter Geometry

Parameters of this spacetime are

- mass, rotational parametr,
- charge,
- cosmological constant, presented by cosmological parameter $y = M^2\Lambda/3$.

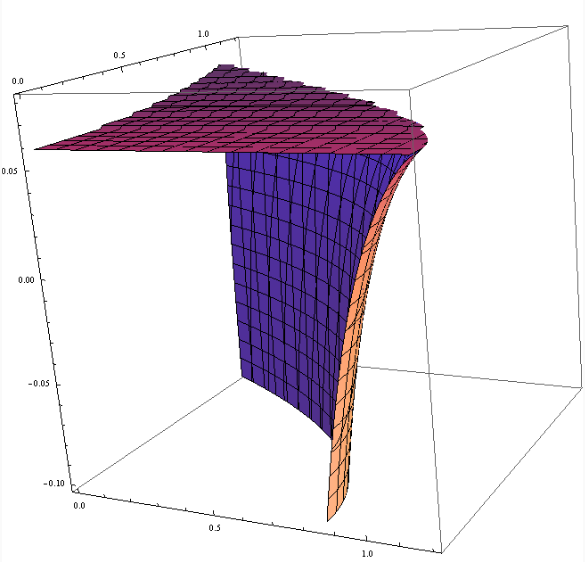
and the Kerr–Newman–de Sitter geometry is given by the line element

$$\mathrm{d}s^2 = -\frac{\Delta_r}{I^2\rho^2}\left(\mathrm{d}t - a\sin^2\theta\,\mathrm{d}\phi\right) + \\ + \frac{\Delta_\theta \sin^2\theta}{I^2\rho^2}[a\,\mathrm{d}t - \left(r^2 + a^2\right)\,\mathrm{d}\phi]^2 + \frac{\rho^2}{\Delta_r}\mathrm{d}r^2 + \frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2\,,$$

$$\Delta_r = \left(r^2 + a^2\right)\left(1 - \frac{1}{3}\Lambda r^2\right) - 2Mr + e^2\,,$$

$$\Delta_\theta = 1 + \frac{1}{3}\Lambda a^2\cos^2\theta\,, \qquad \rho^2 = r^2 + a^2\cos^2\theta\,.$$

Kerr–Newman–de Sitter Geometry



Existence of equatorial circular motion

- The motion of a test particle with rest mass m is given by the geodesic equations.Using the equations obtained by Carter (1973), we can construct an effective potencial and find turning points of the radial motion and finaly, we can derive in the same way as Bardeen (1973) in case of Kerr solution expressions for specific energy and specific angular momentum of circular orbits.

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$$E_{\pm}(r;a,y) = \frac{1+\frac{e^2}{r^2}-\frac{2}{r}-\left(a^2+r^2\right)y \pm a\sqrt{-\frac{e^2}{r^4}+\frac{1}{r^3}-y}}{\sqrt{1+\frac{2e^2}{r^2}-\frac{3}{r}-a^2y \pm 2a\sqrt{-\frac{e^2}{r^4}+\frac{1}{r^3}-y}}},$$

$$L_{\pm}(r;a,y) = \frac{\frac{ae^2}{r^2}-\frac{2a}{r}-ay\left(a^2+r^2\right) \pm \left(a^2+r^2\right)\sqrt{-\frac{e^2}{r^4}+\frac{1}{r^3}-y}}{\sqrt{1+\frac{2e^2}{r^2}-\frac{3}{r}-a^2y \pm 2a\sqrt{-\frac{e^2}{r^4}+\frac{1}{r^3}-y}}}.$$

Equatorial photon orbits

From the second reality condition, we get circular photon orbits. The so called "plus" and "minus" family.

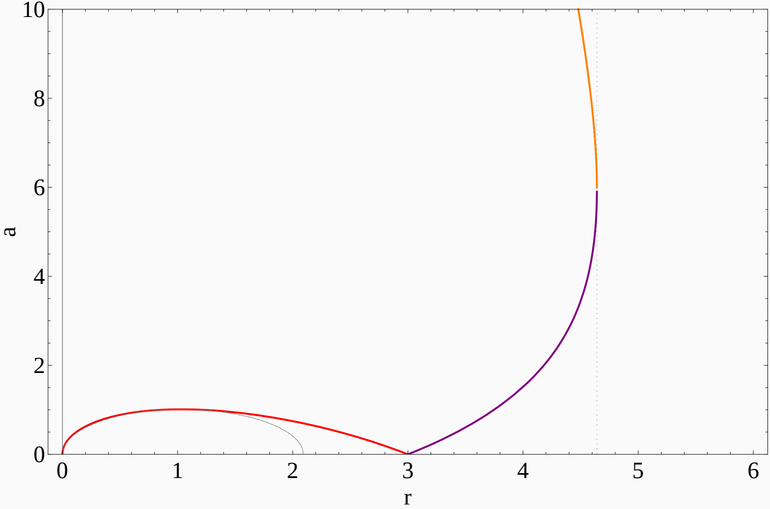
$$a^{(+)}_{ph(1,2)} = \frac{\pm r^2\sqrt{\frac{-e^2+r}{r^4} - y} + \sqrt{r^2\left(2e^2+(-3+r)r\right)y - (e^2 - r + r^4 y)}}{r^2 y},$$

$$a^{(-)}_{ph(1,2)} = \frac{\pm r^2\sqrt{\frac{-e^2+r}{r^4} - y} - \sqrt{r^2\left(2e^2+(-3+r)r\right)y - (e^2 - r + r^4 y)}}{r^2 y}.$$

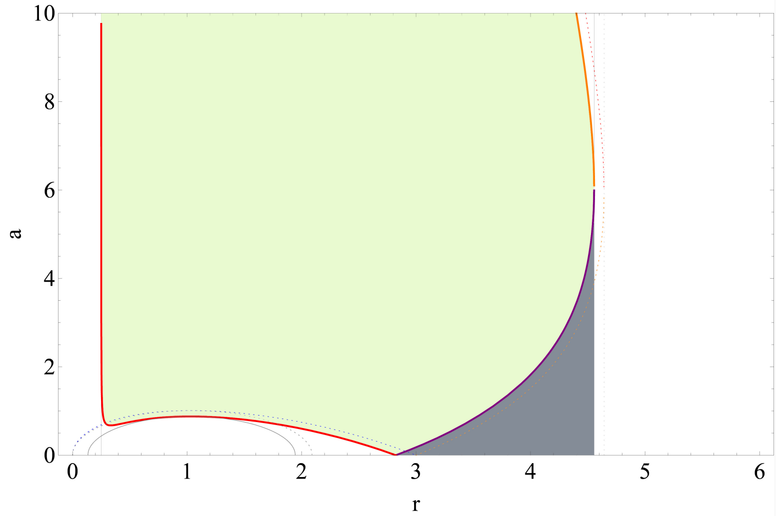
Orientation of rotation

- The behavior of the circular orbits in the field of Kerr black holes ($y = 0$) suggests that the plus-family orbits correspond to the corotating orbits, while the minus-family circular orbits correspond to the counterrotating ones.
- In the Kerr–de Sitter geometry, it is much more complicated, as the geometry is not asymptoticaly flat. We use LNRFs as the measure of orientation of circular orbits and the same process we use in case of the Kerr–Newman–de Sitter geometry, where we are still not finsihed with this calssification.

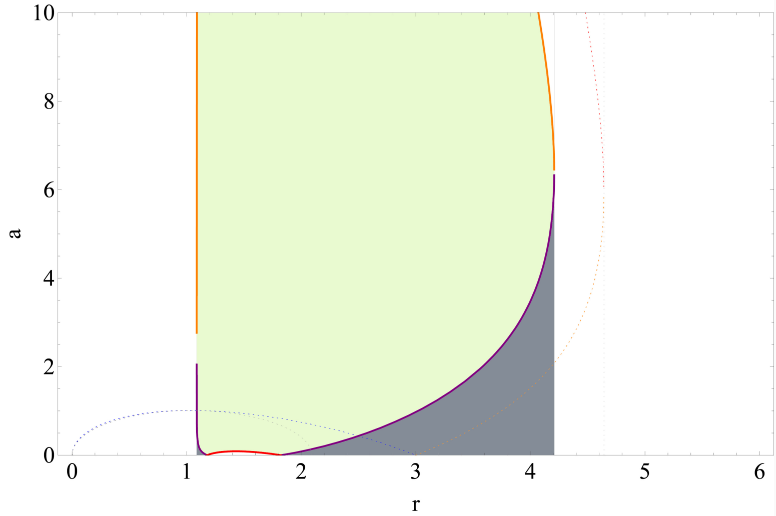
Equatorial circular orbits, $y = 0.01$, $e = 0$



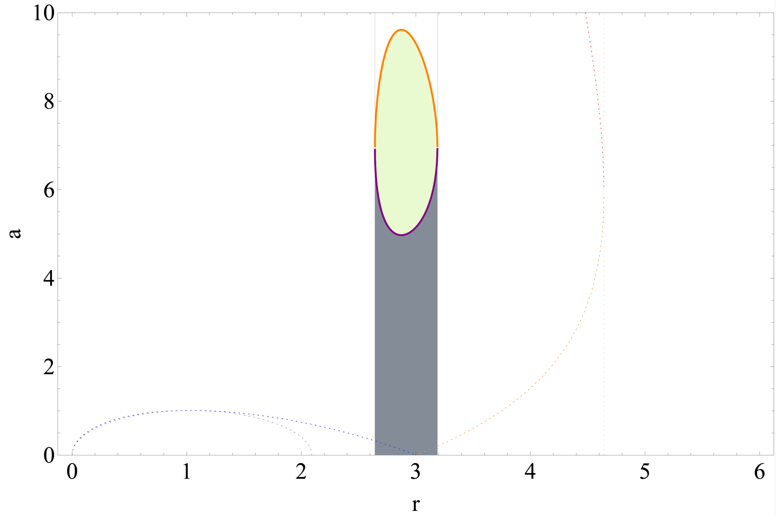
Equatorial circular orbits, $y = 0.01$, $e = 0.5$



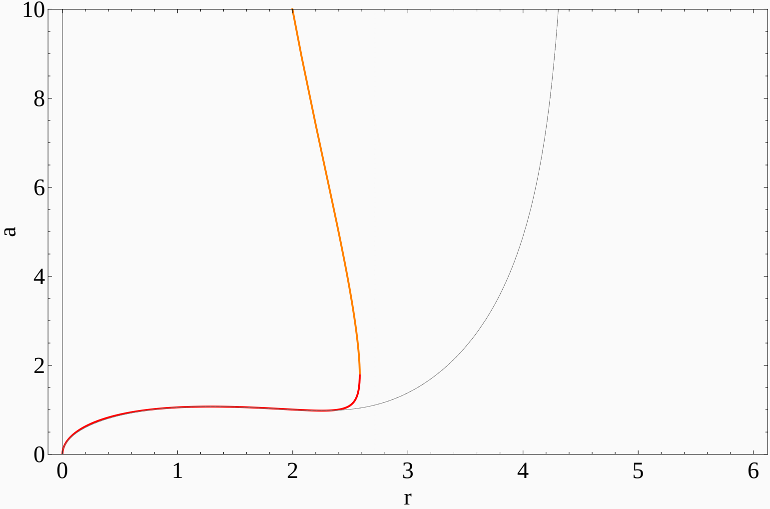
Equatorial circular orbits, $y = 0.01$, $e = 1.036$



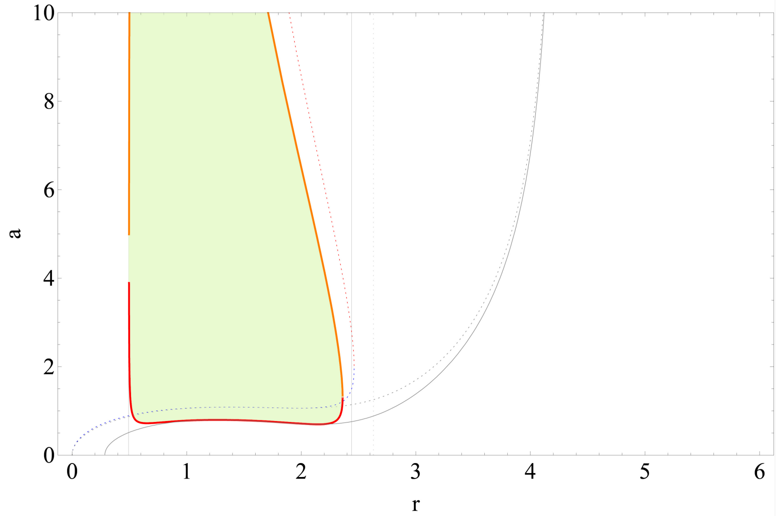
Equatorial circular orbits, $y = 0.01$, $e = 1.468$



Equatorial circular orbits, $y = 0.05$, $e = 0$



Equatorial circular orbits, $y = 0.05$, $e = 0.7$



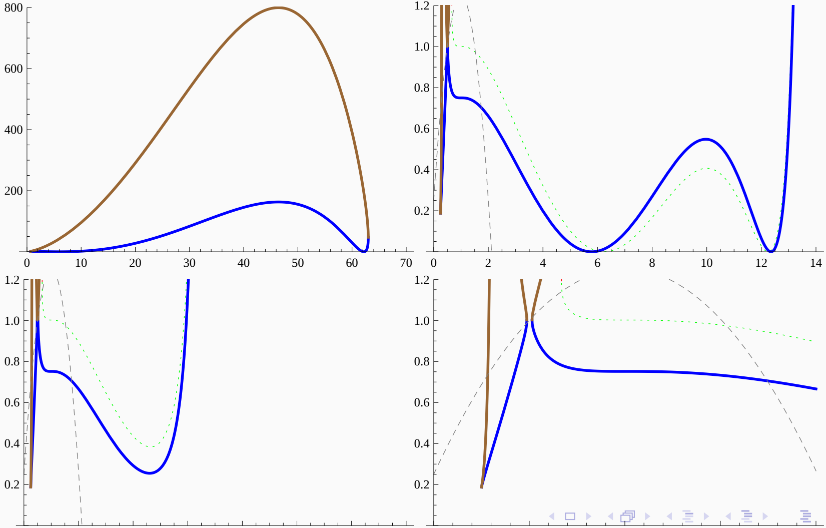
Stability of equatorial circular orbits

The loci of stable circular orbits is given by the condition

$$\frac{\mathrm{d}^2 R}{\mathrm{d}r^2} \geqq 0\,,$$

together with the conditions $R(r) = 0$, $dR(r)/dr = 0$ which determine the specific energy and specific angular momentum of the circular photon orbits. Here we can get quite complicated forms witch can be interpreted in the next plots as marginaly stable sircular orbits and that determines the existence of stable circular orbits. Because of the so many parametrs in this spacetime solution we are not able to directly plot the reality condition for the existence of the stable circular orbits, but we can, at least, plot the most interesting marignaly stable circular orbits.

Stability of circular orbits, NS ($y = 10^{-6}$), BH ($y = 10^{-4}$), BH ($y = 10^{-3}$), BH ($y = 10^{-3}$, zoom)



What to do...

- Make at least some clasification of orientation of circular orbits.
- Finish the reality condition for the existence of the stable circular orbits.

Thank you for your attention.