Self-interacting Fermionic Dark Matter with Axis of Locality ACGRG5

Dimitri Schritt

Collaborators: D.V. Ahluwalia and Cheng-Yang Lee

University of Canterbury, New Zealand

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Motivation

As we all know, the problem of dark matter dates back to the 1930s when Oort and Zwicky postulated the existence of some form of "dark matter" from the motions of celestial bodies.

I will talk about

Dark matter candidate called *Elko*

- fermionic
- spin 1/2
- self interacting
- mass dimension one

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Elko is a very active field

There are now more than 30 publications entirely devoted to Elko ranging from cosmology and astrophysics to mathematics and mathematical physics.

Recent publications on Elko

1. D.V. Ahluwalia, D. Grumiller
   Spin-half Fermions with Mass Dimension One: Theory, Phenomenology, and Dark Matter
   JCAP 0507:012, 2005

2. D.V. Ahluwalia, D. Grumiller
   A Spin one half fermion field with mass dimension one?
   Phys. Rev. D72: 067701, 2005

3. D.V. Ahluwalia, Cheng-Yang Lee, D. Schritt
   Self-interacting Elko dark matter with axis of locality,
   arXiv:0911.2947v1 hep-th

Outline

- Majorana spinors
- Elko
  - spinor properties
  - quantum field

Motivation for Elko

Elko was born out of a desire to understand Majorana spinors and Majorana field.

Majorana spinors defined

Given charge conjugation operator

$$\mathsf{C} = -\gamma^2 \kappa \qquad \text{with} \quad \kappa : \{i \to -i\}$$

Majorana spinors are defined by

$$\mathsf{C}\, \xi(\mathbf{p}) = +1\, \xi(\mathbf{p})$$

Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Gamma matrices:

$$\gamma^i = \begin{pmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{pmatrix} \qquad \gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

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Majorana spinors are defined by

$$\mathsf{C}\,\xi(\mathbf{p}) = +1\,\xi(\mathbf{p})$$

Explicitly we have

$$\xi(\mathbf{p}, \{-,+\}) = \sqrt{m} \begin{pmatrix} +\sigma_2\,\phi_+^*(\mathbf{p}) \\ \phi_+(\mathbf{p}) \end{pmatrix}$$

$$\xi(\mathbf{p}, \{+,-\}) = \sqrt{m} \begin{pmatrix} +\sigma_2\,\phi_-^*(\mathbf{p}) \\ \phi_-(\mathbf{p}) \end{pmatrix}$$

$$\mathbf{J} \cdot \hat{\mathbf{p}}\,\phi_+ = +\tfrac{1}{2}\,\phi_+ \qquad \mathbf{J} \cdot \hat{\mathbf{p}}\,\phi_- = -\tfrac{1}{2}\,\phi_- \qquad \mathbf{J} \equiv (J_i) \equiv \tfrac{1}{2}\,(\sigma_i)$$

Incompleteness of Majorana spinors

Ahluwalia and Grumiller realised that Majorana spinors do not form a complete set.

Charge conjugation matrix γ^2 is 4×4, hence it ought to have four eigenspinors.

The complete set of eigenspinors of C are given by

Self conjugate:

$$\mathsf{C}\,\xi(\mathbf{p}) = +1\,\xi(\mathbf{p})$$

Anti-self conjugate:

$$\mathsf{C}\,\zeta(\mathbf{p}) = -1\,\zeta(\mathbf{p})$$

Majorana spinors (self conjugate) — But Not Grassmann

Elko (self conjugate and anti-self conjugate)

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Self conjugate spinors: Majorana spinors, but not Grassmann. Self conjugate and anti-self conjugate together: Elko.

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$$\left.\begin{array}{ll} \text{Self conjugate:} & \\ & \left.\mathsf{C}\,\xi(\mathbf{p}) = +1\,\xi(\mathbf{p}) \quad \right\} \underbrace{\text{Majorana spinors}}_{\text{But Not Grassmann}} \\ \text{Anti-self conjugate:} & \\ & \mathsf{C}\,\zeta(\mathbf{p}) = -1\,\zeta(\mathbf{p}) \end{array}\right\} \text{Elko}$$

Elko

So as to avoid confusion with the Majorana spinors, Ahluwalia and Grumiller coined ELKO:

Eigenspinoren des Ladungskonjugationsoperators

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Eigenspinoren des **L**adungs**k**onjugations**o**perators

Explicitly, they are given by:

$$\xi(\mathbf{0}, \{-,+\}) = \sqrt{m} \begin{pmatrix} +\sigma_2 \, \phi_+^*(\mathbf{0}) \\ \phi_+(\mathbf{0}) \end{pmatrix}$$

$$\xi(\mathbf{0}, \{+,-\}) = \sqrt{m} \begin{pmatrix} +\sigma_2 \, \phi_-^*(\mathbf{0}) \\ \phi_-(\mathbf{0}) \end{pmatrix}$$

$$\zeta(\mathbf{0}, \{-,+\}) = \sqrt{m} \begin{pmatrix} -\sigma_2 \, \phi_-^*(\mathbf{0}) \\ \phi_-(\mathbf{0}) \end{pmatrix}$$

$$\zeta(\mathbf{0}, \{+,-\}) = \sqrt{m} \begin{pmatrix} -\sigma_2 \, \phi_+^*(\mathbf{0}) \\ \phi_+(\mathbf{0}) \end{pmatrix}$$

Here ϕ_+ and ϕ_- are Weyl spinors. The '+' and '−' denote the spin sign of the projections.

Weyl Spinors

In the *polarisation basis* the underlying Weyl spinors read

$$\phi_+(\mathbf{0}) = \sqrt{m}\begin{pmatrix} \mathrm{e}^{-i\phi/2} \\ 0 \end{pmatrix}$$

$$\phi_-(\mathbf{0}) = \sqrt{m}\begin{pmatrix} 0 \\ \mathrm{e}^{+i\phi/2} \end{pmatrix}$$

Where we define

$$\mathbf{0} \equiv \frac{\mathbf{p}}{\|\mathbf{p}\|}\Bigg|_{\|\mathbf{p}\|\to 0}$$

Elko in Polarisation Basis

Self conjugate Elko at rest:

$$\xi(\mathbf{0},\{-,+\}) = \sqrt{m}\begin{pmatrix} 0 \\ i\mathrm{e}^{+i\phi/2} \\ \mathrm{e}^{-i\phi/2} \\ 0 \end{pmatrix}$$

$$\xi(\mathbf{0},\{+,-\}) = \sqrt{m}\begin{pmatrix} -i\mathrm{e}^{-i\phi/2} \\ 0 \\ 0 \\ \mathrm{e}^{+i\phi/2} \end{pmatrix}$$

Anti-self conjugate Elko at rest:

$$\zeta(\mathbf{0},\{-,+\}) = \sqrt{m}\begin{pmatrix} i\mathrm{e}^{-i\phi/2} \\ 0 \\ 0 \\ \mathrm{e}^{+i\phi/2} \end{pmatrix}$$

$$\zeta(\mathbf{0},\{+,-\}) = \sqrt{m}\begin{pmatrix} 0 \\ i\mathrm{e}^{+i\phi/2} \\ -\mathrm{e}^{-i\phi/2} \\ 0 \end{pmatrix}$$

Elko in Polarisation Basis

Elko spinors at momentum $\mathbf{p}$:

$$
\begin{aligned}
\xi(\mathbf{p},\{-,+\}) &= \mathsf{B}(\mathbf{p})\,\xi(\mathbf{0},\{-,+\}) \\
\xi(\mathbf{p},\{+,-\}) &= \mathsf{B}(\mathbf{p})\,\xi(\mathbf{0},\{+,-\}) \\
\zeta(\mathbf{p},\{-,+\}) &= \mathsf{B}(\mathbf{p})\,\zeta(\mathbf{0},\{-,+\}) \\
\zeta(\mathbf{p},\{+,-\}) &= \mathsf{B}(\mathbf{p})\,\zeta(\mathbf{0},\{+,-\})
\end{aligned}
$$

where

$$\mathsf{B}(\mathbf{p}) = \kappa_r \oplus \kappa_l$$

$$\kappa_{r/l} \equiv \exp\left(\pm\frac{\boldsymbol{\sigma}}{2}\cdot\boldsymbol{\varphi}\right) = \sqrt{\frac{E+m}{2m}}\left(\mathbb{1} \pm \frac{\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}}{E+m}\right)$$

Elko do not satisfy Dirac equation

Unlike Dirac spinors, Majorana spinors and Elko do not satisfy the Dirac equation:

$$\begin{aligned}
\gamma_\mu p^\mu\, \xi(\mathbf{p}, \{-,+\}) &= +im\, \xi(\mathbf{p}, \{+,-\}) \\
\gamma_\mu p^\mu\, \xi(\mathbf{p}, \{+,-\}) &= -im\, \xi(\mathbf{p}, \{-,+\}) \\
\gamma_\mu p^\mu\, \zeta(\mathbf{p}, \{-,+\}) &= -im\, \zeta(\mathbf{p}, \{+,-\}) \\
\gamma_\mu p^\mu\, \zeta(\mathbf{p}, \{+,-\}) &= +im\, \zeta(\mathbf{p}, \{-,+\})
\end{aligned}$$

$$\Longrightarrow \qquad (\gamma_\mu p^\mu - m)\, \chi(\mathbf{p}) \neq 0 \quad \chi \in \{\xi_{\{-,+\}}, \xi_{\{+,-\}}, \zeta_{\{-,+\}}, \zeta_{\{+,-\}}\}$$

Elko adjoint

Ahluwalia and Grumiller introduced the following adjoint for Elko:

$$\chi(\mathbf{p}, \{\mp, \pm\}) \quad \longrightarrow \quad \overset{\neg}{\chi}(\mathbf{p}, \{\mp, \pm\}) \equiv \mp i \left[\chi(\mathbf{p}, \pm, \mp)\right]^{\dagger} \gamma_0$$

The norms then become:

$$\overset{\neg}{\xi}(\mathbf{p}, \alpha)\, \xi(\mathbf{p}, \alpha') = +2m\delta_{\alpha\alpha'}$$

$$\overset{\neg}{\zeta}(\mathbf{p}, \alpha)\, \zeta(\mathbf{p}, \alpha') = -2m\delta_{\alpha\alpha'}$$

where $\alpha \in \{\{-, +\}, \{+, -\}\}$.

Elko completeness relation

$$\frac{1}{2m}\sum_{\alpha}\left[\xi(\mathbf{p},\alpha)\,\stackrel{\neg}{\xi}(\mathbf{p},\alpha)-\zeta(\mathbf{p},\alpha)\,\stackrel{\neg}{\zeta}(\mathbf{p},\alpha)\right]=\mathbb{1}$$

Elko spin sums

$$\sum_\alpha \xi(\mathbf{p},\alpha)\, \overset{\neg}{\xi}\,(\mathbf{p},\alpha) = m\,[\mathcal{G}(\phi) + \mathbb{1}]$$

$$\sum_\alpha \zeta(\mathbf{p},\alpha)\, \overset{\neg}{\zeta}\,(\mathbf{p},\alpha) = m\,[\mathcal{G}(\phi) - \mathbb{1}]$$

where

$$\mathcal{G}(\phi) = i \begin{pmatrix} 0 & 0 & 0 & -\mathrm{e}^{-i\phi} \\ 0 & 0 & \mathrm{e}^{i\phi} & 0 \\ 0 & -\mathrm{e}^{-i\phi} & 0 & 0 \\ \mathrm{e}^{i\phi} & 0 & 0 & 0 \end{pmatrix}$$

Dirac counterpart:

$$\sum_\alpha u(\mathbf{p},\alpha)\bar{u}(\mathbf{p},\alpha) = m\left[\frac{\gamma_\mu p^\mu}{m} + \mathbb{1}\right]$$

$$\sum_\alpha v(\mathbf{p},\alpha)\bar{v}(\mathbf{p},\alpha) = m\left[\frac{\gamma_\mu p^\mu}{m} - \mathbb{1}\right]$$

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This already suggests that a Elko cannot have mass dimension 3/2.

Elko Fermionic Fields

Can one construct a local quantum field with Elko expansion coefficients?

"Dirac type" with distinct particle and anti-particle:

$$\Lambda(x) = \int \frac{d^3p}{\sqrt{2mE}} \sum_\alpha \left[\mathrm{e}^{-ip\cdot x} \xi(\mathbf{p},\alpha) a(\mathbf{p},\alpha) + \mathrm{e}^{+ip\cdot x} \zeta(\mathbf{p},\alpha) b^\ddagger(\mathbf{p},\alpha) \right]$$

$$\overset{\neg}{\Lambda}(x) = \int \frac{d^3p}{\sqrt{2mE}} \sum_\alpha \left[\mathrm{e}^{+ip\cdot x} \, \overset{\neg}{\xi}(\mathbf{p},\alpha) a^\ddagger(\mathbf{p},\alpha) + \mathrm{e}^{-ip\cdot x} \, \overset{\neg}{\zeta}(\mathbf{p},\alpha) b(\mathbf{p},\alpha) \right]$$

Elko Fermionic Fields

“Majorana type” with particle of same type as anti-particle:

$$\lambda(x) = \int \frac{d^3p}{\sqrt{2mE}} \sum_{\alpha} \left[\mathrm{e}^{-ip\cdot x} \xi(\mathbf{p},\alpha) a(\mathbf{p},\alpha) + \mathrm{e}^{+ip\cdot x} \zeta(\mathbf{p},\alpha) a^{\ddagger}(\mathbf{p},\alpha) \right]$$

$$\stackrel{\neg}{\lambda}(x) = \int \frac{d^3p}{\sqrt{2mE}} \sum_{\alpha} \left[\mathrm{e}^{+ip\cdot x}\ \stackrel{\neg}{\xi}(\mathbf{p},\alpha) a^{\ddagger}(\mathbf{p},\alpha) + \mathrm{e}^{-ip\cdot x}\ \stackrel{\neg}{\zeta}(\mathbf{p},\alpha) a(\mathbf{p},\alpha) \right]$$

Propagator

The fermionic propagator is proportional to

$$S(x - x') = \langle \ |\mathfrak{T}[\Lambda(x)\ \bar{\Lambda}\ (x')]|\ \rangle$$

where the time ordered product is defined by

$$\mathfrak{T}[\Lambda(x)\ \bar{\Lambda}(x')] \equiv \begin{cases} +\ \Lambda(x)\ \bar{\Lambda}(x') & \text{if } \ x_0 > x_0' \\ -\ \bar{\Lambda}(x')\ \Lambda(x) & \text{if } \ x_0' > x_0 \end{cases}$$

Propagator and Preferred Axis

$$S(x-x') = i\int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-x')}\Big[\frac{\mathbb{1}+\mathcal{G}(\phi)}{q\cdot q - m^2 + i\epsilon}\Big]$$

Choosing a preferred axis s.t. $\mathbf{x}-\mathbf{x}'$ is along the $\hat{z}$ direction, $q\cdot(x-x')$ is rendered independent of θ, and we obtain

$$\int d^4q\,\mathcal{G}(\phi) = 0$$

Once the above preferred axis is chosen, Elko is endowed with a Klein Gordon propagator

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Field Equation

The Klein Gordon equation is the corresponding Green's function

$$(\partial^\mu \partial_\mu + m^2) S(x) = -\delta^4(x)$$

and thereby the wave equation for Elko:

$$(\partial_\mu \partial^\mu + m^2)\Lambda = (\partial_\mu \partial^\mu + m^2)\lambda = 0$$

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Lagrangian Density and Mass Dimensionality

The corresponding Lagrangian densities are

$$\mathcal{L}_{\Lambda}(x) = \partial^{\mu} \overset{\neg}{\Lambda}(x)\, \partial_{\mu}\Lambda(x) - m^2\, \overset{\neg}{\Lambda}(x)\, \Lambda(x)$$

$$\mathcal{L}_{\lambda}(x) = \partial^{\mu} \overset{\neg}{\lambda}(x)\, \partial_{\mu}\lambda(x) - m^2\, \overset{\neg}{\lambda}(x)\, \lambda(x)$$

In order for the theory to be renormalisable, we must have:

$$[\mathcal{L}] = [M]^4$$

Thus the quantum fields must be of mass dimension one

$$[\Lambda] = [\lambda] = [M]$$

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Self-interaction

Terms of the form $\stackrel{\neg}{\Lambda}\Lambda\stackrel{\neg}{\Lambda}\Lambda$ are of mass dimension one, hence Elko enjoys an unsuppressed self-interaction.

Darkness

Due to the relative mass dimensionalities

- Elko quantum fields: $[\Lambda] = [\lambda] = [M]$
- Dirac quantum field: $[\Psi] = [M]^{3/2}$

terms of the form $\bar{\Psi}\,\Psi\,\neg{\Lambda}\,\Lambda$ are suppressed by one order of unification scale

$$\frac{1}{M_P}\left[\bar{\Psi}\,\Psi\,\overset{\neg}{\Lambda}\,\Lambda\right] = [M]^4$$

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Locality

There exists an axis defined by $\hat{z}$ in which the locality anticommutators for Elko are given by:

$$\{\Lambda(\mathbf{x},t),\Lambda^{\ddagger}(\mathbf{x}',t)\} = \{\lambda(\mathbf{x},t),\lambda^{\ddagger}(\mathbf{x}',t)\} = 0$$

$$\{\Lambda(\mathbf{x},t),\Lambda(\mathbf{x}',t)\} = \{\lambda(\mathbf{x},t),\lambda(\mathbf{x}',t)\} = 0$$

$$\{\Pi(\mathbf{x},t),\Pi(\mathbf{x}',t)\} = \{\pi(\mathbf{x}),\pi(\mathbf{x}',t)\} = 0$$

$$\{\Lambda(\mathbf{x},t),\Pi(\mathbf{x}',t)\} = \{\lambda(\mathbf{x},t),\pi(\mathbf{x}',t)\} = i\delta^3(\mathbf{x}-\mathbf{x}')$$

We consequently call this the *axis of locality*.

Summary

- Field constructed out of Elko spinors
  - is local
  - is fermionic
  - is of spin 1/2
  - is of mass dimension one $\Rightarrow$ DARK
  - is self interacting
  - has an axis of locality (may or may not exist in dark sector c.f. "Axis of Evil")

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