Encoding Cosmological Futures with Conformal Structures

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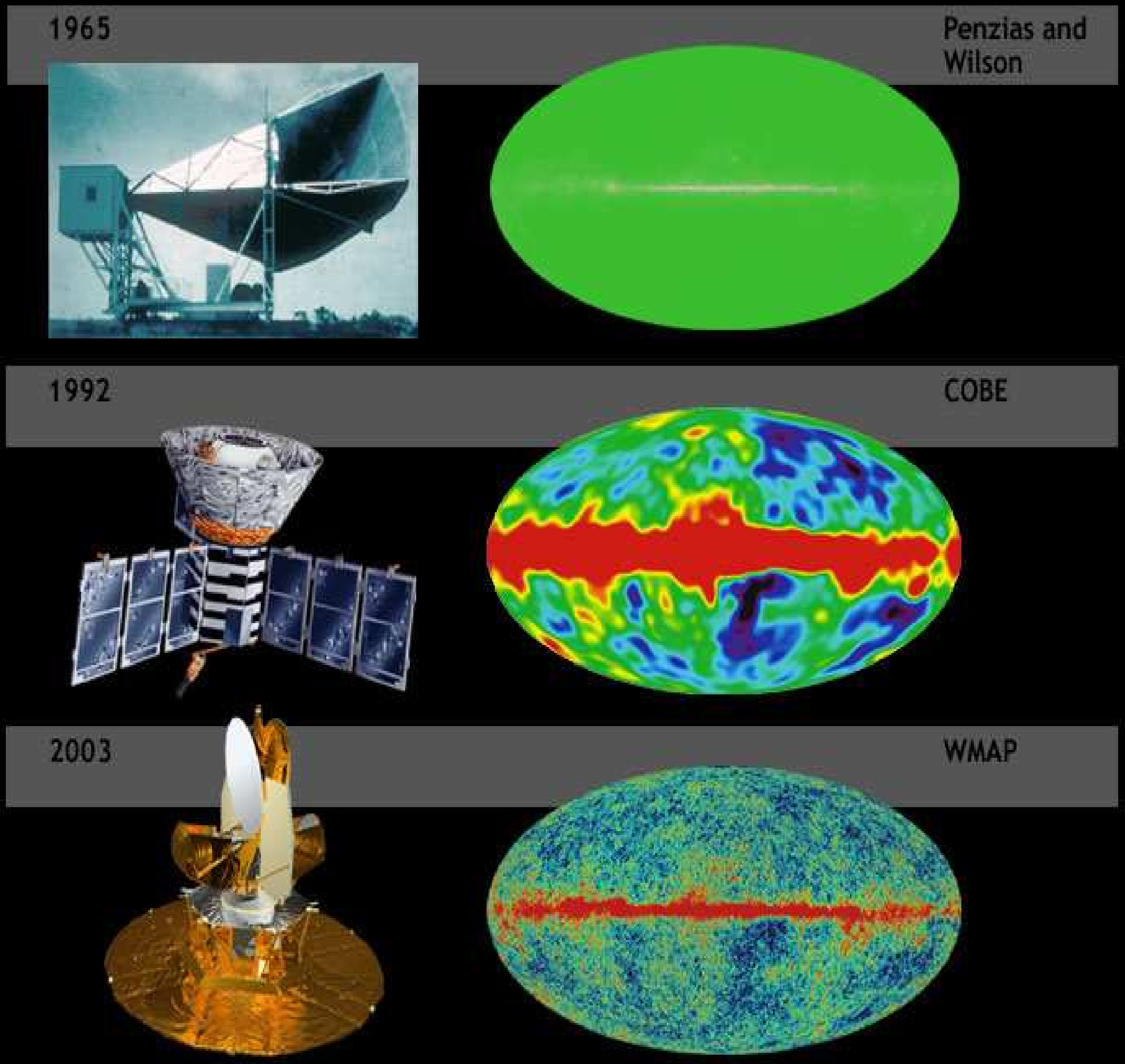
Outline

1. Background: Isotropy, Quiescent Cosmology...
2. Framework of an Isotropic Past Singularity
3. Motivation for a framework for cosmological futures
4. New definitions and their implications

1. Background: Isotropy, Quiescent Cosmology...

Observations: universe is isotropic in our observable vicinity



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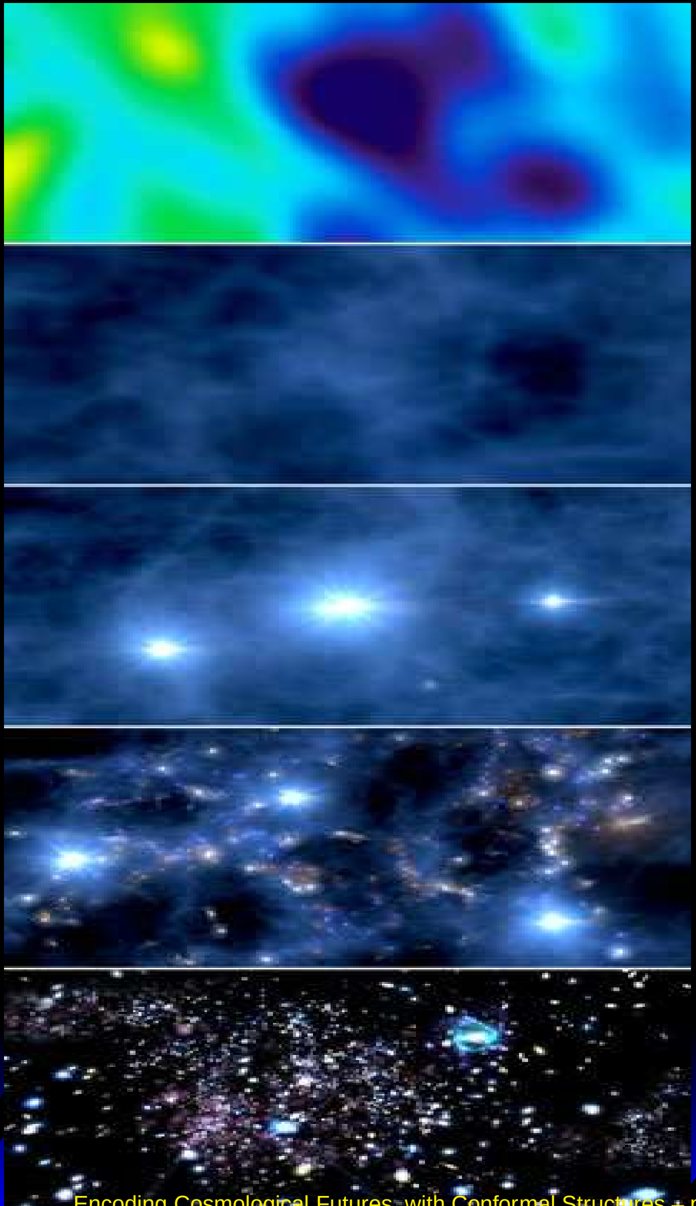
- however: problems with dissipation and entropy

Alternative

Quiescent Cosmology (Barrow 1978)

- lack of chaos in infant universe
  $\Rightarrow$ initially matter in thermal equil.
  $\Rightarrow$ entropy apparently high

- nevertheless: entropy rise due to anomalous behaviour of gravitational entropy $\Rightarrow$ increases with clumping $\Rightarrow$ maximal in a black hole (maximally clumpy object) $S_{BH} \propto M^2$



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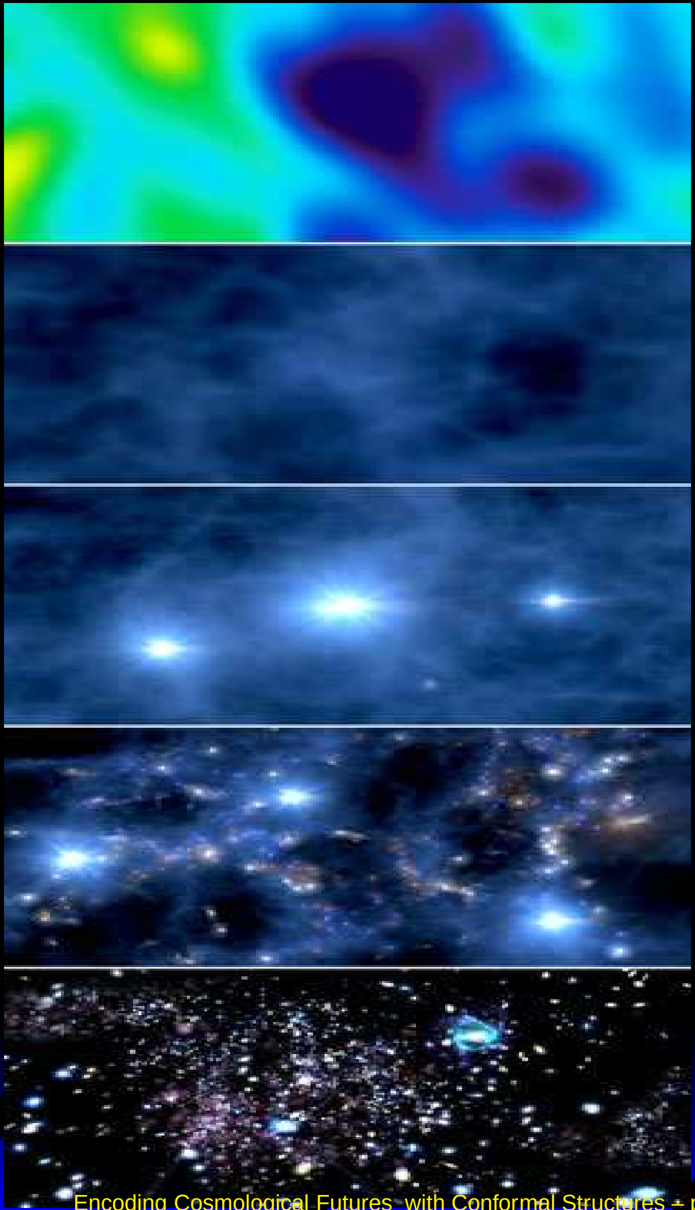

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hypothesis for the initial low-entropy constraint (Penrose 1979)

grav. entropy $\Leftrightarrow$ grav. clumping $\Leftrightarrow$ Weyl tensor C_{abcd}

Weyl curvature is bounded, e.g. matter dominated

$$\lim_{T \to 0^+} \frac{C_{abcd}C^{abcd}}{R_{ef}R^{ef}} = 0.$$

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- universe appears isotropic, because we happen to live at an *early stage* of its evolution

Summary of quiescent cosmology for the initial state

- thermal equilibrium and C_{abcd} bounded

$\Rightarrow$ spatial isotropy and homogeneity at *Big Bang*

$\Rightarrow$ initial singularity must have been "isotropic" as in the FRW models

Cosmological fluid flow and kinematics

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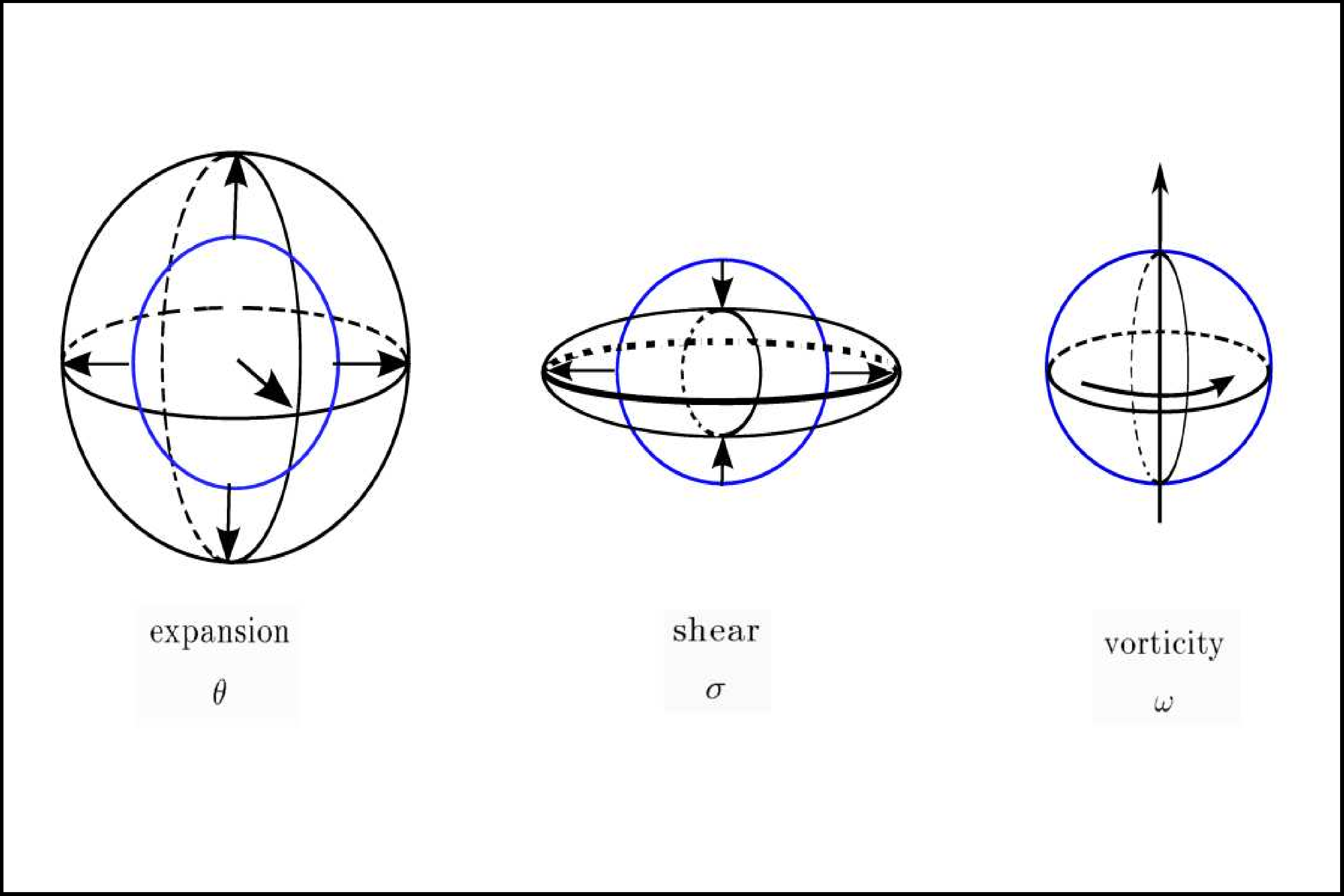
- define kinematic quantities - expansion θ, shear σ, vorticity ω and acceleration $\dot{u}^a := u^a{}_{;b} u^b$ - via covar. deriv. of $\mathbf{u}$

expansion θ, shear σ and vorticity ω of the fluid



calculated via derivatives of $\mathbf{u}$

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(1) *Kinematic isotropy relative to $\mathbf{u}$:*

no preferred directions due to shear, vorticity and acceleration ($\sigma = \omega = 0$, $\dot{u}^a = 0$)

(2) *Weyl isotropy:*

$C_{abcd} \equiv 0 \Rightarrow$ no principal null directions

2. The framework of the Isotropic Past Singularity

Motivation: structure of an example model

Kantowski-Sachs models: irrotational, geodesic perfect fluid cosmologies with radiation equation of state $p = \frac{1}{3}\mu$

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metric given by

$$ds^2 = -A(t)dt^2 + t\left[A^{-1}(t)dx^2 + A^2(t)b^{-2}\left(dy^2 + \sin^2 y dz^2\right)\right]$$

where

$$A(t) = 1 - \frac{4b^2t}{9}, \quad t > 0 \text{ and } b = const$$

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$$\begin{aligned} ds^2 &= T^2\left[-A(T)dT^2 + A^{-1}(T)d\tilde{x}^2 + A^2(T)\tilde{b}^{-2}\left(dy^2 + \sin^2 y dz^2\right)\right] \\ &= \Omega^2(T)d\tilde{s}^2 \end{aligned}$$

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- $ds^2 \to 0$ as $T \to 0^+$, but $d\tilde{s}^2$ completely regular at $T = 0 \Rightarrow$ singularity of physical ds^2 absorbed in $\Omega(0) = 0$

Isotropic Past Singularity (IPS)

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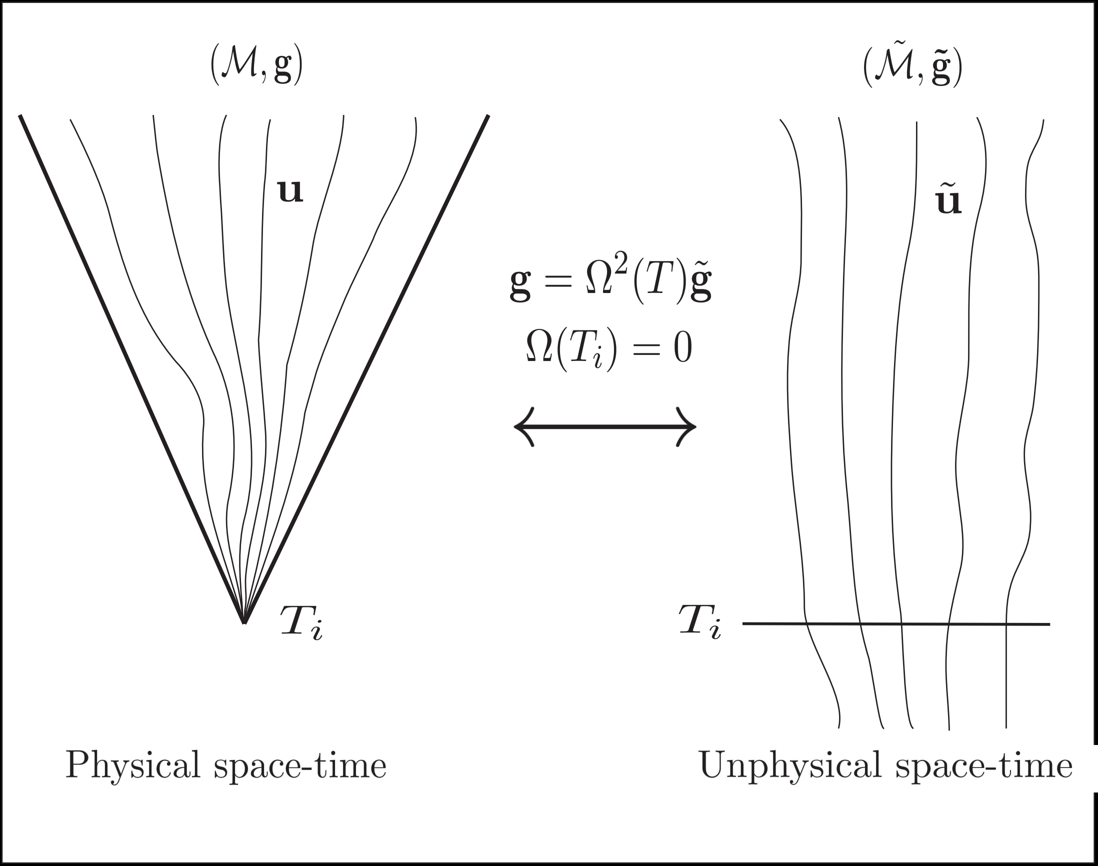
- Definition of an *IPS* relates phys. space-time $(\mathcal{M}, \mathrm{g})$ to unphys. space-time $(\tilde{\mathcal{M}}, \tilde{\mathrm{g}})$ via conformal structure $\mathrm{g} = \Omega^2 (T) \, \tilde{\mathrm{g}}$, with
  1. T cosmic time function on $(\tilde{\mathcal{M}}, \tilde{\mathrm{g}})$ and $\mathcal{M}$ open submanifold $T > 0$.
  2. regularity condition for $\tilde{\mathrm{g}}$ on $T = 0$,
  3. $\Omega\,(0) = 0 \Rightarrow$ causes *IPS* at $T = 0$, and
  4. some general constraints on $\Omega(T)$.

Isotropic Past Singularity

- trick: move singular behaviour into $\Omega(T)$ $\Rightarrow$ regularity of $\tilde{\mathrm{g}}$ facilitates analysis of physical quantities at IPS $\Rightarrow$ analytical advantage



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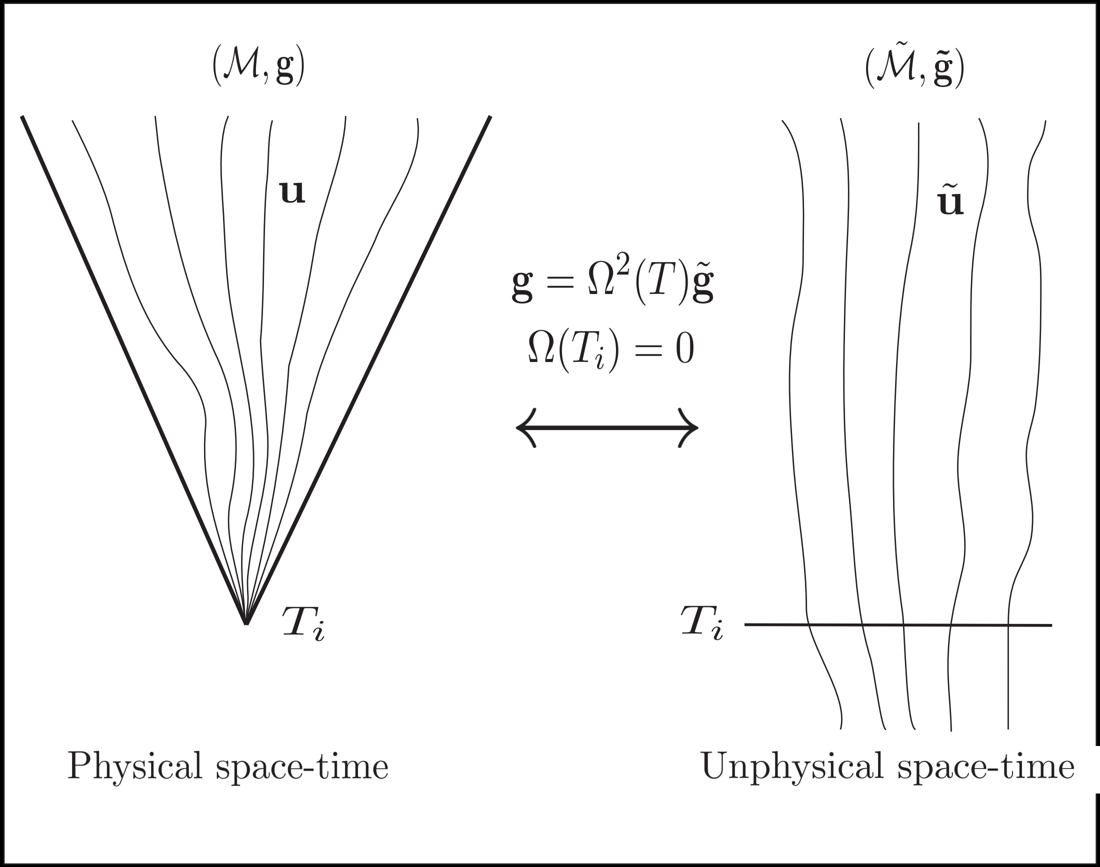
- among other implications, definition guarantees asymptotic initial isotropy and compatibilty with the WCH



$$\lim_{T\to 0^+} \frac{C_{abcd}C^{abcd}}{R_{ef}R^{ef}} = \lim_{T\to 0^+} \frac{\sigma^2}{\theta^2} = \lim_{T\to 0^+} \frac{\omega^2}{\theta^2} = \lim_{T\to 0^+} \frac{\dot{u}^a \dot{u}_a}{\theta^2} = 0$$

3. Motivation for new definitions

Future evolution?

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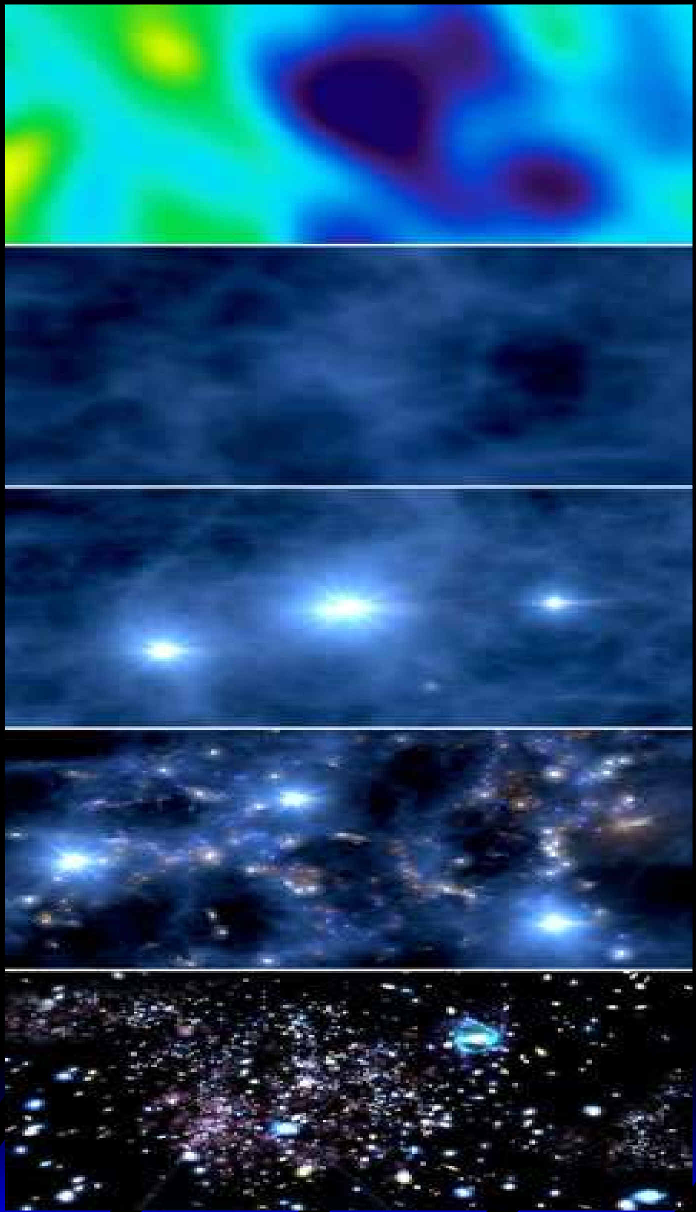

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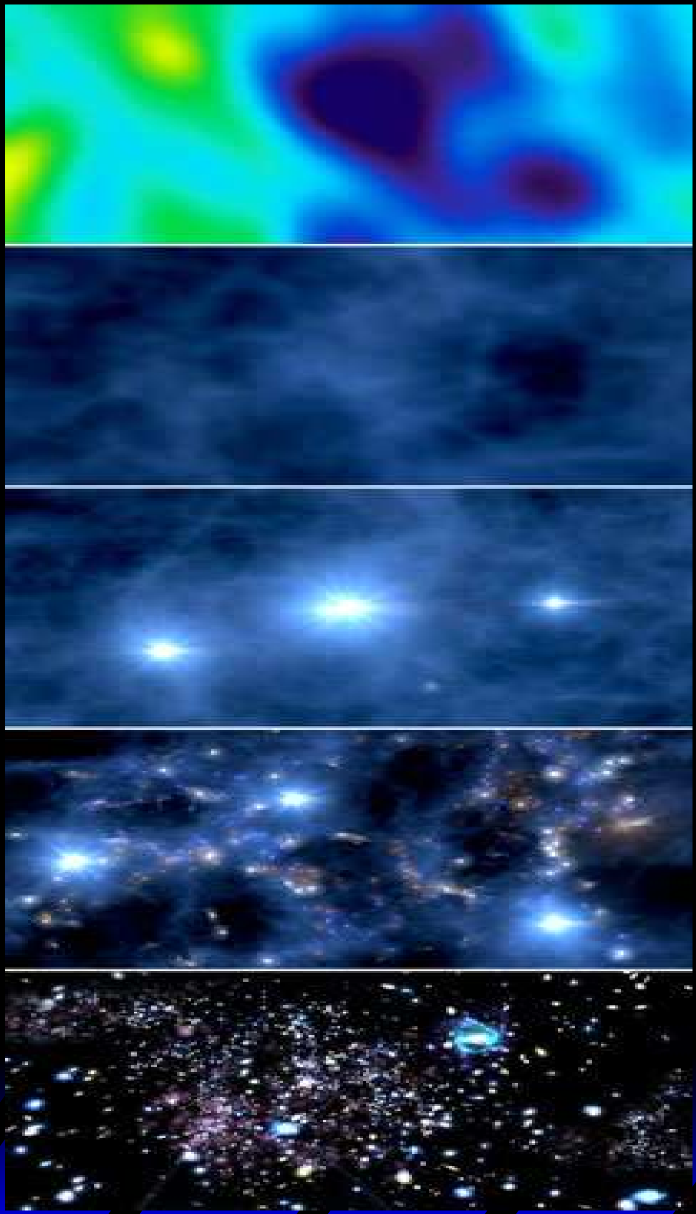
$\Rightarrow$ IPS framework not sufficient to guarantee an anisotropic future evolution $\Rightarrow$ complementary framework necessary



Regular conformal structures and (an)isotropy

- proved 2 theorems:
  conformal structures with regular conformal metric necessarily lead to asymptotic isotropy (valid for $\Omega \to 0$ or $\Omega \to \infty$ and $T \to 0^+$ or $T \to 0^-$):

$$\lim_{T\to 0^\pm} \frac{C_{abcd}C^{abcd}}{R_{ef}R^{ef}} = \lim_{T\to 0^\pm} \frac{\sigma^2}{\theta^2} = \lim_{T\to 0^\pm} \frac{\omega^2}{\theta^2} = \lim_{T\to 0^\pm} \frac{\dot{u}^a \dot{u}_a}{\theta^2} = 0$$

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- important conclusion and justification of IPS definition:
  if conformal structure $\mathrm{g} = \Omega^2\,(T)\,\tilde{\mathrm{g}}$ used in cosmology, then
  1. regular conformal relation for the (isotropic) initial state
  2. irregular conformal relation for the (anisotropic) final state

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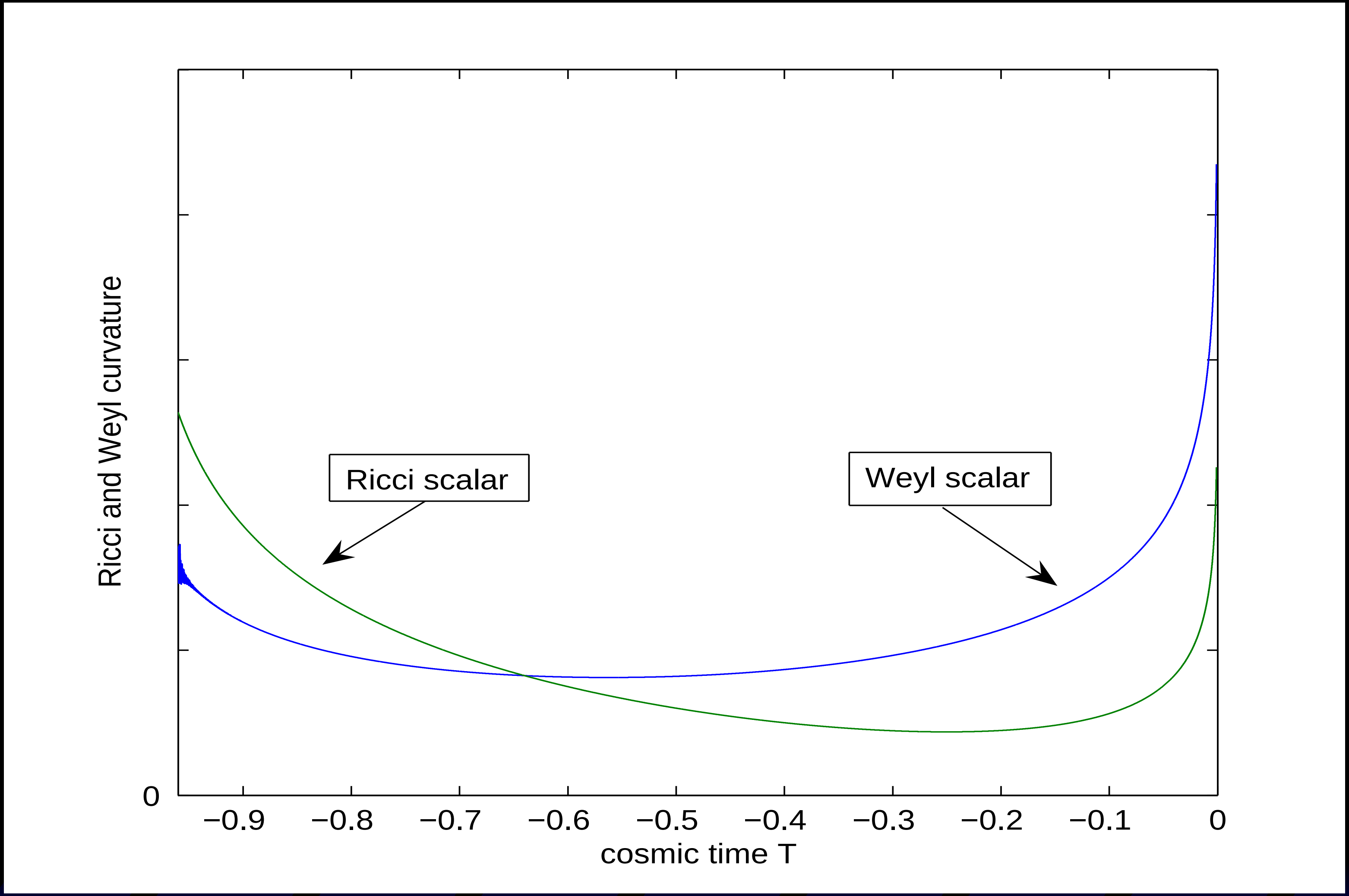
- ... admit anisotropic future behaviour

$$\lim_{\bar{T}\to 0^-} \frac{\sigma}{\theta} = o(1)$$

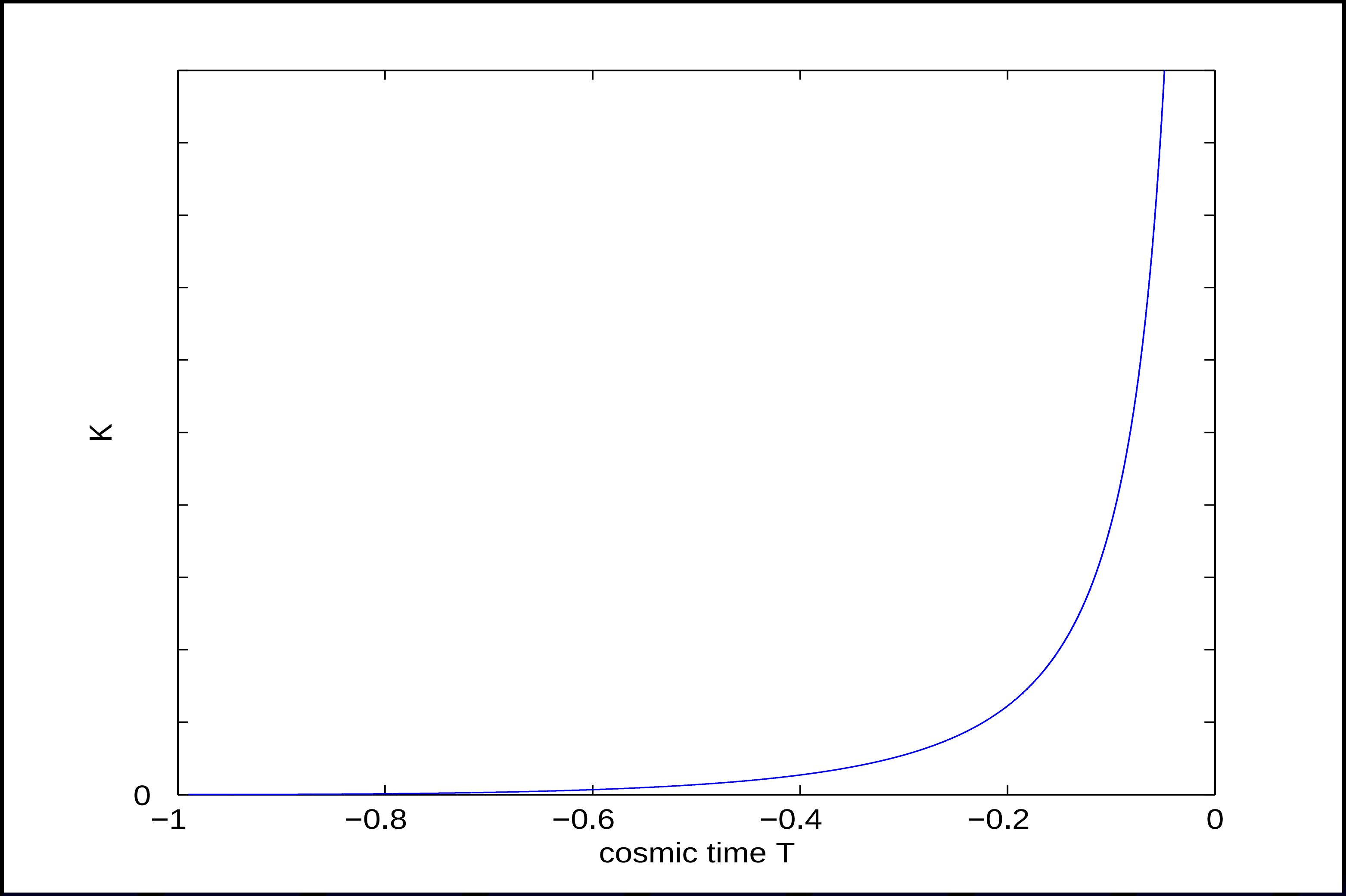
and...

Evolution of the curvature invariants, $R_{ab}R^{ab}$ and $C_{abcd}C^{abcd}$, in the Kantowski-Sachs models until the end of (a rescaled) cosmic time



(logarithmic plot, the y-axis labels are unessential for our purposes and have been omitted).

Evolution of the anisotropy-("gravitational entropy")-measure, $K = C_{abcd}C^{abcd}/R_{ab}R^{ab}$, in the Kantowski-Sachs models



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Conformal structure for a cosmological future in the Kantowski-Sachs models

metric given by (comov. coords.)

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- choose a cosmic time function $\bar{T}$ s.t. $\bar{T} \in (-1, 0)$
  $\bar{T} = -A^2 = -\left(1 - \frac{4b^2 t}{9}\right)^2$

Kantowski-Sachs 2

- rewriting the metric and factoring out a common factor yields

$$ds^2 = \frac{1}{\sqrt{-\bar{T}}}\left[-\frac{81}{64b^4}d\bar{T}^2 + \left(1-\sqrt{-\bar{T}}\right)\right.$$
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- 1. $\bar{\Omega} \to +\infty$, as $\bar{T} \to 0^-$ (new)

  2. $\bar{\mathbf{g}}$ possesses vanishing determinant at $\bar{T} = 0$
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- confirms earlier theorems and influences new definition

4. New definitions and some implications

Prerequisites

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$$F^+(\mathcal{M}) := \{p \in \bar{\mathcal{M}} \supset \mathcal{M} \mid \exists \text{ a future inextendible causal curve } \gamma_p(s) : [0, a) \to \mathcal{M}, \text{ where } a \in \mathbb{R}^+ \cup \{\infty\}, \text{ s. t. } p = \gamma_p(a) \equiv \lim_{s \to a} \gamma_p(s)\}$$

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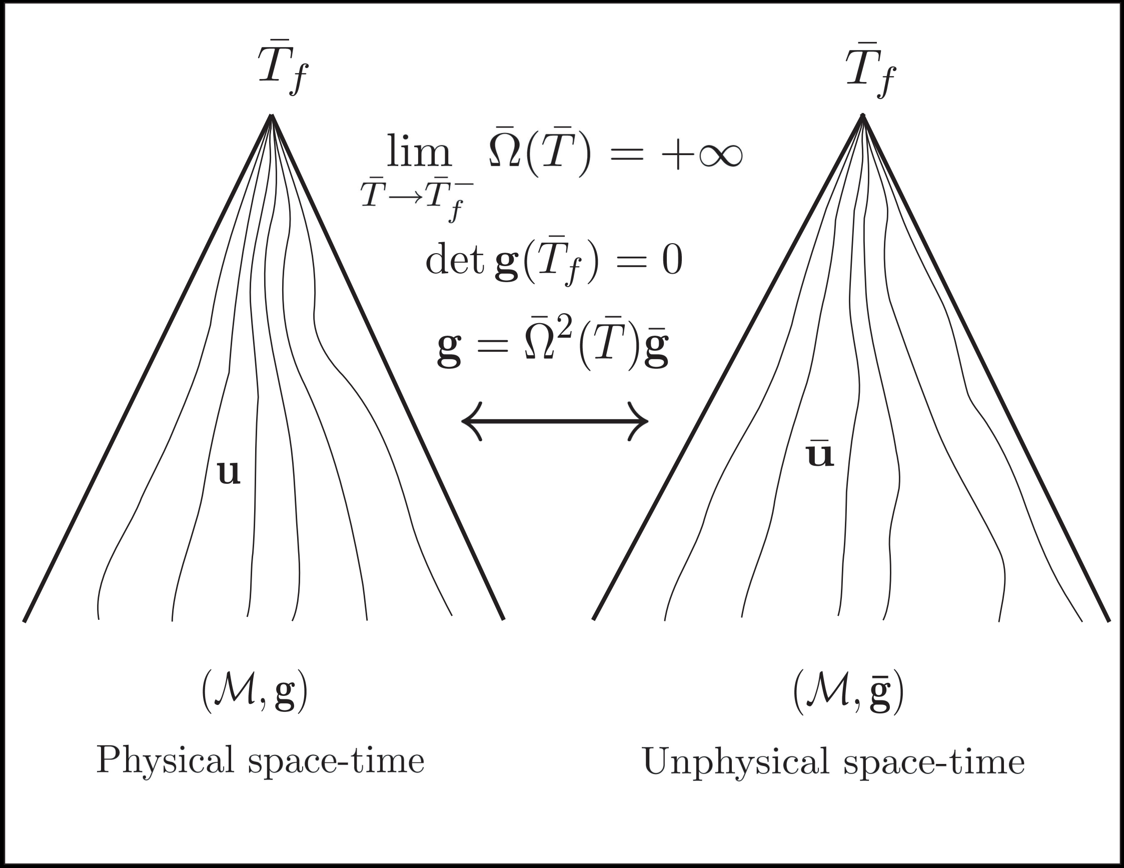
- define *causal degeneracy*: The metric g is said to be *causally degenerate* at $p \in F^+(\mathcal{M})$ if $\exists$ causal curve γ_p with $\gamma_p' \neq 0$ which satisfies $\mathrm{g}(\gamma_p', X) = 0 \; \forall \; X \in T_p\bar{\mathcal{M}}$

Anisotropic Future Singularity (AFS)

- Definition of an *AFS* relates phys. space-time $(\mathcal{M}, \mathrm{g})$ to unphys. space-time $(\mathcal{M}, \bar{\mathrm{g}})$ via conformal structure $\mathrm{g} = \bar{\Omega}^2 \left(\bar{T}\right) \bar{\mathrm{g}}$, furthermore:
  1. $\bar{T}$ defined on $\mathcal{M} \cup F^+(\mathcal{M}) \subset \bar{\mathcal{M}}$, $\bar{T}$ cosmic time funtion on $\mathcal{M}$ with range $\bar{T} < 0$, and $\bar{T} = 0$ on $F^+(\mathcal{M})$,
  2. $\bar{\mathrm{g}}$ degenerate, but not causally degenerate on $F^+(\mathcal{M})$, but $\bar{\mathrm{g}}$ remains C^0,
  3. $\lim_{\bar{T} \to 0^-} \bar{\Omega}\left(\bar{T}\right) = +\infty$, and other general constraints on $\bar{\Omega}$,
  4. $\lim_{\bar{T} \to 0^-} \det(\bar{\Omega}^2 \bar{\mathbf{g}}) = 0$

Anisotropic future singularity

- $\det \mathbf{g}(0) = \det \bar{\mathbf{g}}(0) = 0$ causes singularity for both $(\mathcal{M}, \mathrm{g})$ and $(\mathcal{M}, \bar{\mathrm{g}})$



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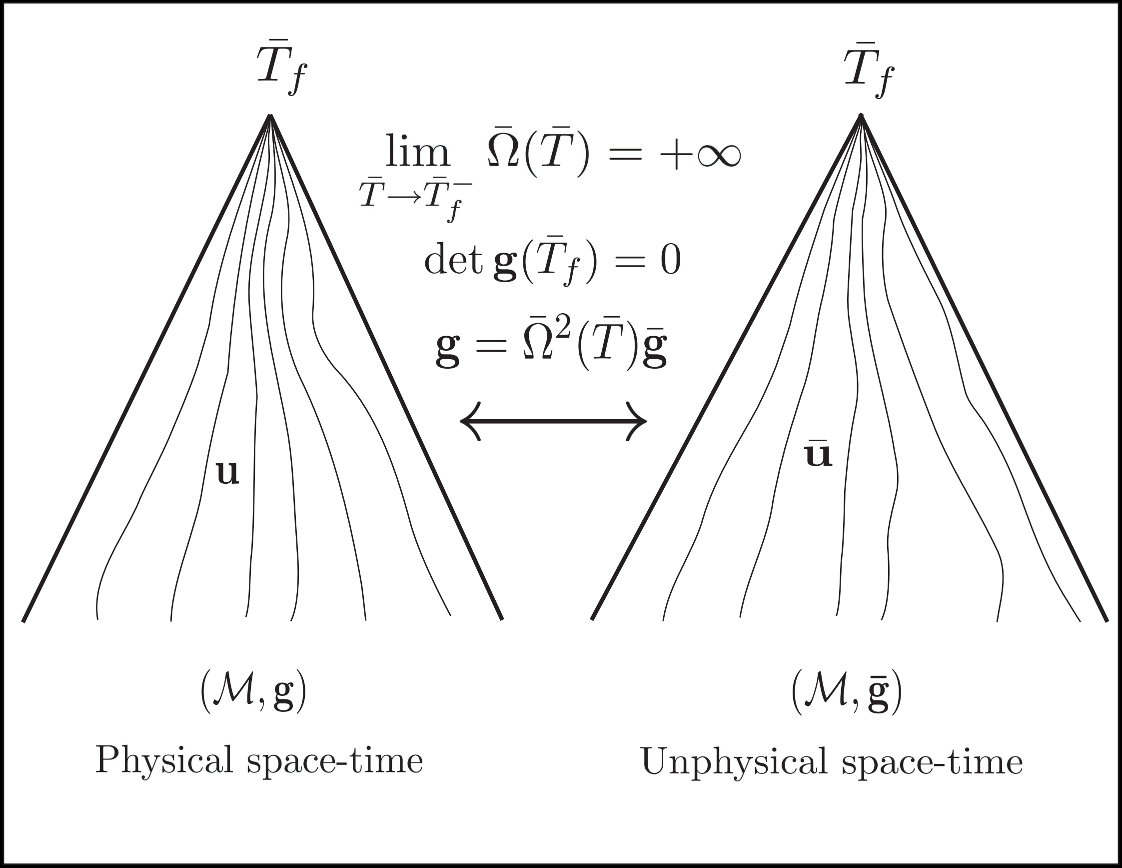
- however: conformal space-time "more regular" than physical space-time
  $\Rightarrow$ analytically helpful



Other example cosmologies, some characteristics

Model	future singularity?	anisotropic future	$\bar{\Omega}(0)$	regular $\bar{\mathbf{g}}$?	$\det \mathbf{g}(0)$
rad. FRW ($k = +1$)	yes	no	0	yes	0
dust FRW ($k = +1$)	yes	no	0	yes	0
Szekeres (subclass)	no	yes	$+\infty$	no	$+\infty$
Mars (3rd type)	no	yes	$+\infty$	no	$+\infty$
Carneiro-Marugan	no	yes	$+\infty$	no	$+\infty$
Kantowski	no	yes	$+\infty$	no	$+\infty$
Kantowski-Sachs	yes	yes	$+\infty$	no	0

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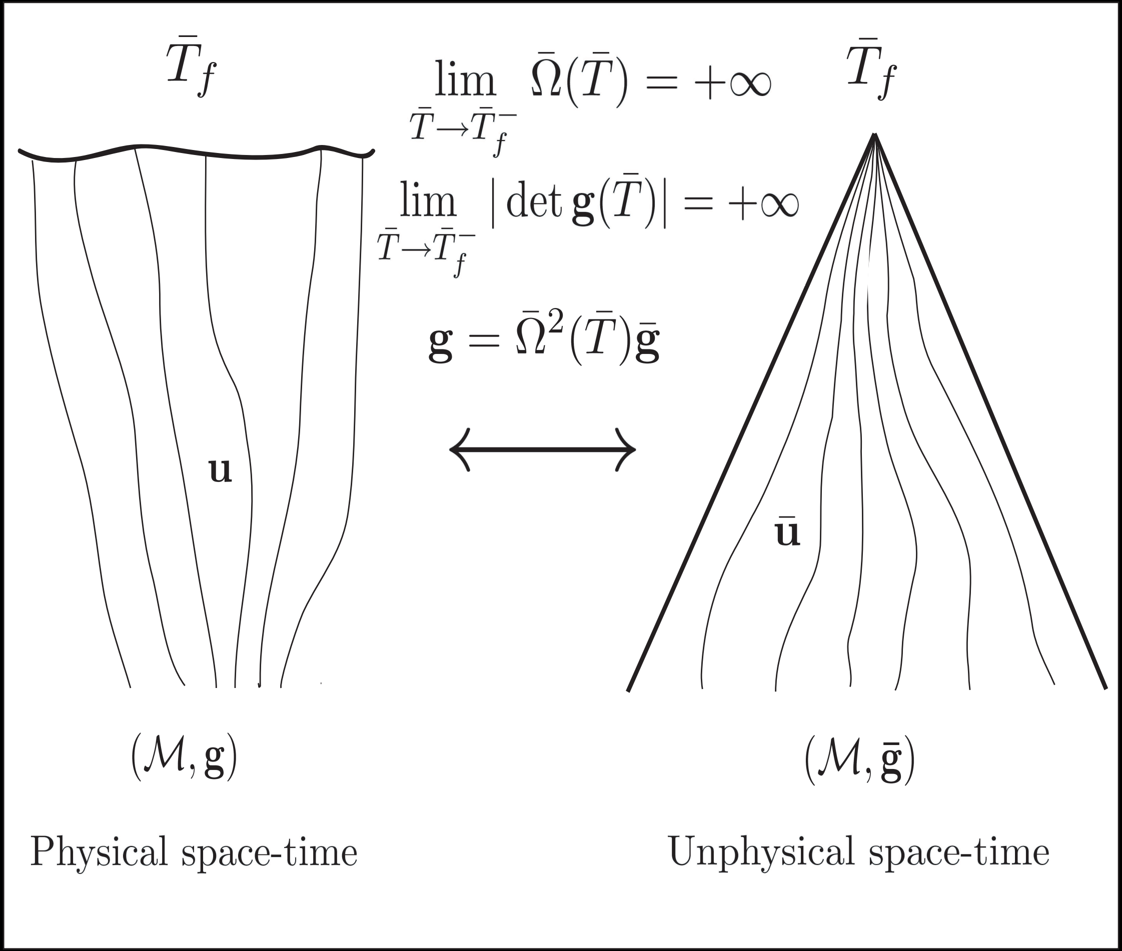
$\Rightarrow$ there should exist other definitions $\Rightarrow$ need definition for anisotropic ever expanding unverse

Anisotropic future endless universe (AFEU)

- same conditions as for AFS, except that now

$$\det(\bar{\Omega}^2 \bar{\mathbf{g}}) \to \infty$$



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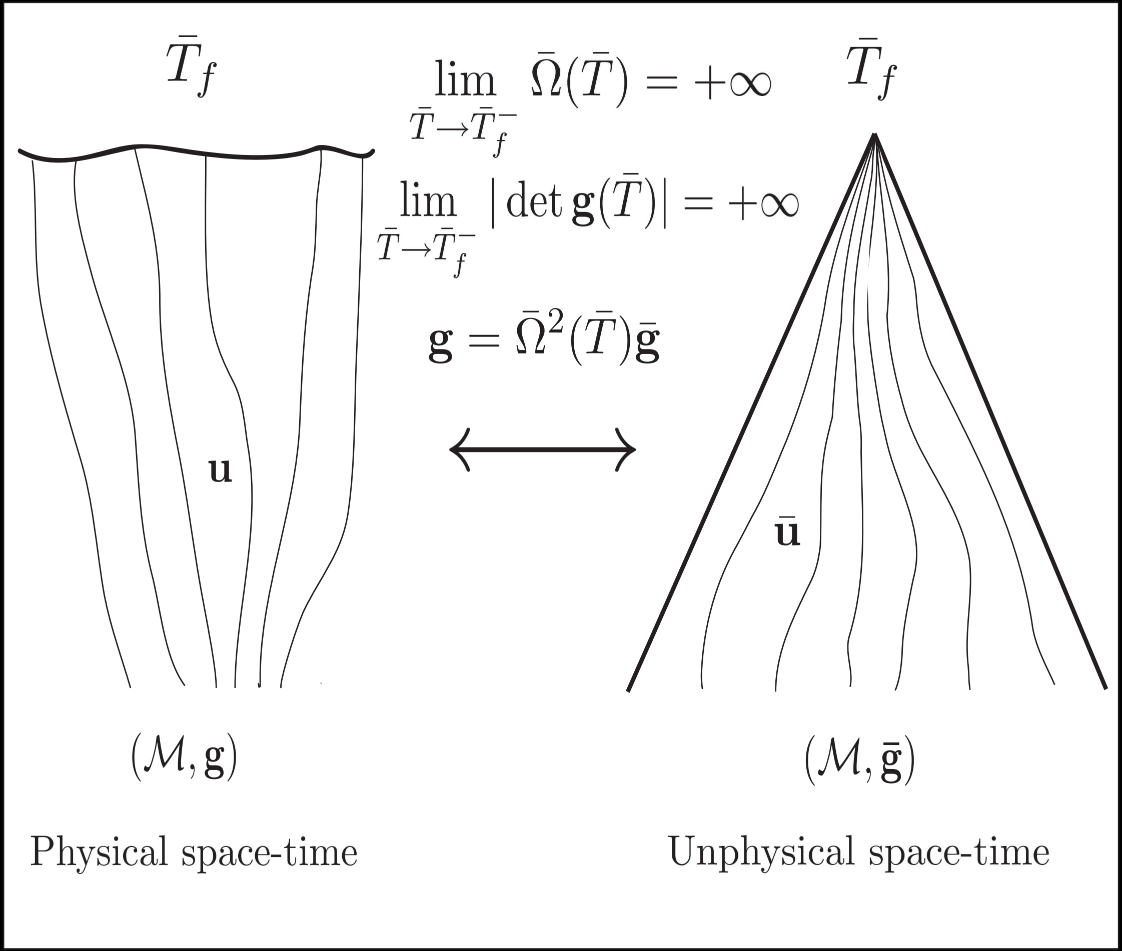

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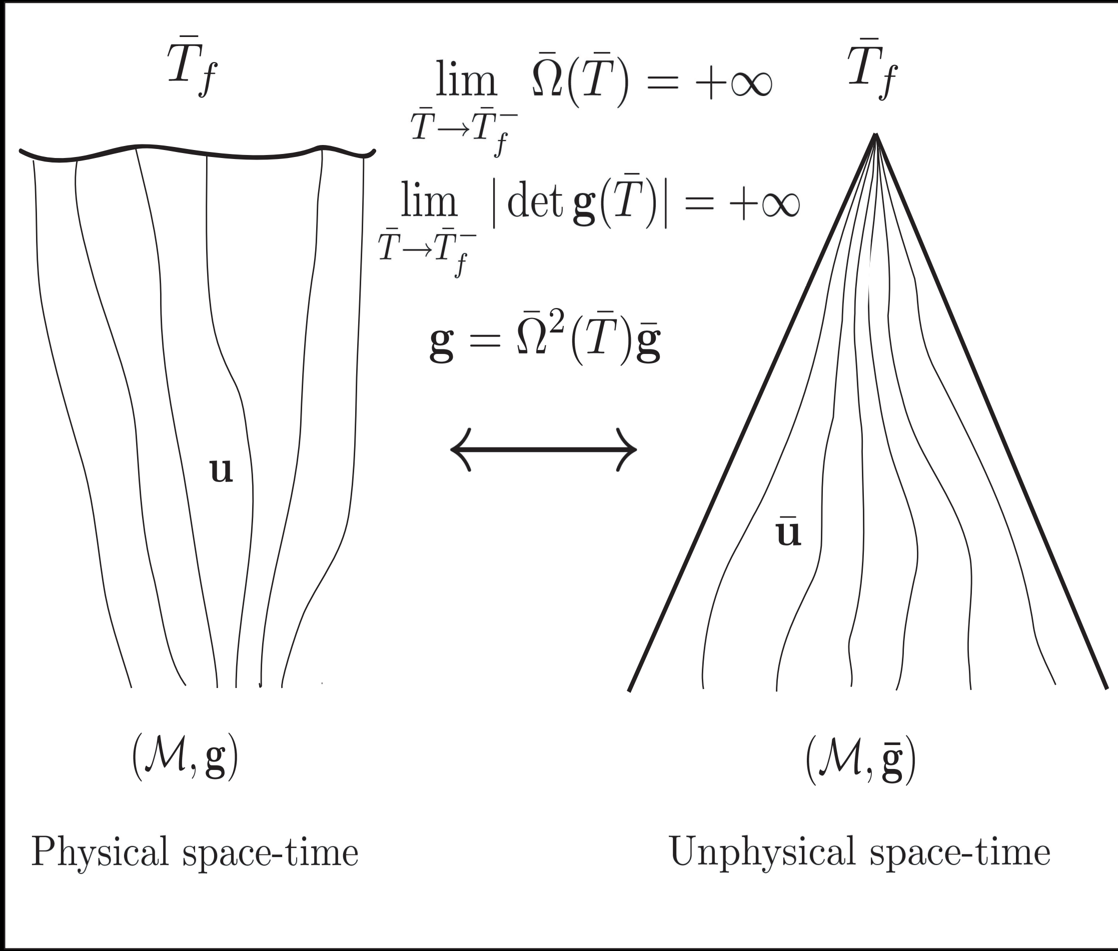
- $\det(\bar{\Omega}^2 \bar{\mathbf{g}}) \to \infty$ causes diverging 4-volume form for $(\mathcal{M}, \mathbf{g})$, while $\det \bar{\mathbf{g}}(0) = 0$ causes singularity for $(\mathcal{M}, \bar{\mathbf{g}})$
- *no physical singularity for physical space-time, but singularity for conformal space-time*



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- analysed example cosmologies show:
  both definitions admit a great variety of (anisotropic) future behaviours

Strong Curvature and Jacobi fields

- Jacobi fields J^i represent displacement vectors between neighbouring geodesics and satisfy *geodesic deviation eqn.*

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- definition of a *Tipler Strong Curvature Singularity* (TSCS) requires physical objects to be crushed to zero:
  $\liminf_{s \rightarrow t_s} V(s) = 0$
  $\Rightarrow$ end of space-time

Strong Curvature at an AFS (IPS)

- using continuity and degeneracy condition of $\bar{\mathrm{g}}$ on $F^+(\mathcal{M})$
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- similar theorems show for the first time:
  IPS is a TSCS for <u>causal</u> geodesics

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Conclusion

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- theorems $\Rightarrow$ emphasise IPS and imply irregular conformal structure for anisotropy
- analysed example cosmologies as guidance
- provided new framework via defs. of the AFS and the AFEU
- derived some phys. implications (e.g. θ, strong curvature...)
- <u>conjecture:</u> combination of the IPS with the AFS and the AFEU provides first version of a complete formalisation of Quiescent Cosmology

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- etc ...

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