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Geodesics in impulsive vacuum spacetimes with a cosmological constant

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Spacetimes of constant curvature

- Spacetime is *conformally flat* when its line element can be written in the form

$$g_{ab} = \Omega^2 \eta_{ab} \ , \tag{1}$$

  where η_{ab} is the flat metric and Ω is conformal factor.

- Spacetime is of *constant curvature* when the Riemman tensor everywhere satisfies

$$R_{abcd} = \tfrac{1}{12} R (g_{ac} g_{bd} - g_{ad} g_{bc}) \ ,$$

  whence it follows that Ricci scalar is $R = 4\Lambda$.

According to the signum of the cosmological constant Λ we can distinguish

- *Minkowski spacetime:* $\Lambda = 0$
- *de Sitter spacetime:* $\Lambda > 0$
- *anti-de Sitter spacetime:* $\Lambda < 0$

We can write these spacetimes in the form (1), namely

$$\mathrm{d}s^2 = \frac{-\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2}{\left[1 + \frac{\Lambda}{12}(-t^2 + x^2 + y^2 + z^2)\right]^2} \ , \qquad \text{or equivalently} \qquad \mathrm{d}s^2 = \frac{2\mathrm{d}\zeta \mathrm{d}\bar{\zeta} - 2\mathrm{d}\mathcal{U}\mathrm{d}\mathcal{V}}{[1 + \frac{1}{6}\Lambda(\zeta\bar{\zeta} - \mathcal{U}\mathcal{V})]^2} \ .$$

(anti-)de Sitter spacetime

We may visualize (anti-)de Sitter spacetime as a *4-dimensional hyperboloid*

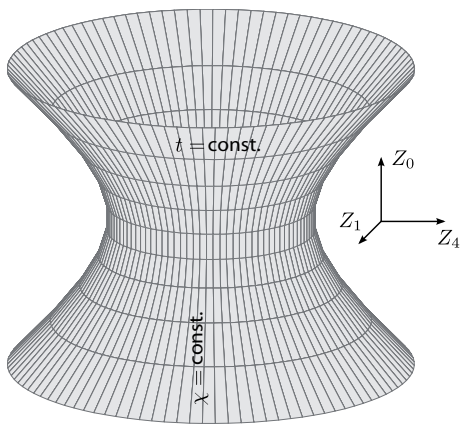
$$-Z_0^2 + Z_1^2 + Z_2^2 + Z_3^2 + \sigma Z_4^2 = \sigma a^2 \ , \tag{2}$$

in flat 5-dimensional space

$$\mathrm{d}s^2 = -\mathrm{d}{Z_0}^2 + \mathrm{d}{Z_1}^2 + \mathrm{d}{Z_2}^2 + \mathrm{d}{Z_3}^2 + \sigma \mathrm{d}{Z_4}^2 \ ,$$

where $a = \sqrt{\frac{3}{|\Lambda|}}$, and σ is signum of cosmological constant Λ.

For $\Lambda > 0$, the de Sitter hyperboloid (2) is naturaly spanned by t, χ, θ and ϕ



Metric in these coordinates takes the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2 \cosh^2 \frac{t}{a} \left[\mathrm{d}\chi^2 + \sin^2 \chi (\mathrm{d}\theta^2 + \sin^2 \theta \mathrm{d}\phi^2) \right] \ ,$$

where $t \in (-\infty, +\infty)$, $\chi \in (0, \pi)$, $\theta \in (0, \pi)$ and $\phi \in [0, 2\pi)$ (the section $t = const.$ is a 3-sphere).
Worldlines with constant χ, θ and ϕ are timelike geodesics.

Impulsive spherical gravitational waves

Expanding impulsive spherical gravitational waves can be constructed by the *Penrose "cut and paste" method* [Penrose (1972)], i. e. "cutting" a spacetime along a null cone and "putting it together" with a suitable warp:

$$\left[Z, \bar{Z}, V, U = 0_-\right]_{\mathcal{M}_-} \equiv \left[h(Z), \bar{h}(\bar{Z}), \frac{(1+\epsilon h\bar{h})V}{(1+\epsilon Z\bar{Z})|h'|}, U = 0_+\right]_{\mathcal{M}_+} . \tag{3}$$

The impulse is located on the hypersurface $U = 0$, which is a sphere expanding with the speed of light $(x^2 + y^2 + z^2 = t^2)$.

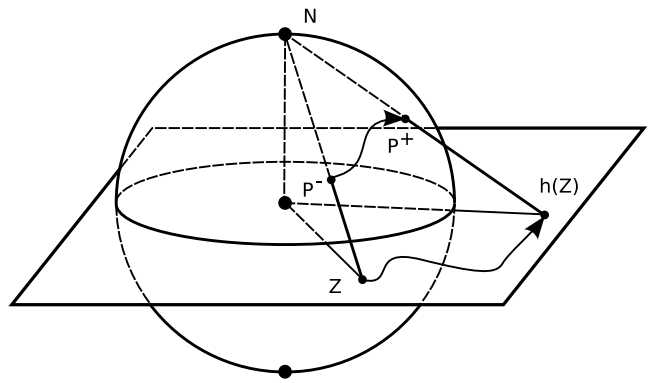


Figure 1: Geometrical interpretation of the Penrose junction conditions (3): mapping in the complex plane $Z \to h(Z)$ gives a mapping of points on the Riemann sphere from $P^- \to P^+$.

Continuous metric which describes such waves is
[Hogan (1992, 1993, 1994), Podolský and Griffiths (1999)]

$$\mathrm{d}s^2 = \frac{2\left|\frac{V}{p}\mathrm{d}Z + U\Theta\left(U\right)p\bar{H}\mathrm{d}\bar{Z}\right|^2 + 2\mathrm{d}U\mathrm{d}V - 2\epsilon\mathrm{d}U^2}{\left[1 + \frac{1}{6}\Lambda U(V - \epsilon U)\right]^2} ,$$

where

$$p = 1 + \epsilon Z\bar{Z} , \qquad \epsilon = -1, 0, +1 .$$

Relation of the continuous coordinates to the conformally flat coordinates of the background

- *behind the impulse*: $U < 0$

$$\mathcal{V}^- = \frac{V}{p} - \epsilon U \ , \qquad \mathcal{U}^- = \frac{Z\bar{Z}}{p} V - U \ , \qquad \zeta^- = \frac{Z}{p} V \ .$$

- *in front of the impulse*: $U > 0$

$$\begin{aligned} \mathcal{V}^+ &= AV - DU \ , \\ \mathcal{U}^+ &= BV - EU \ , \\ \zeta^+ &= CV - FU \ , \end{aligned}$$

where the parameters A, B, C, D, E a F are functions of Z resp. $\bar{Z}$, namely

$$\begin{aligned} A &= \frac{1}{p|h'|} \ , \qquad B = \frac{|h|^2}{p|h'|} \ , \qquad C = \frac{h}{p|h'|} \ , \\ D &= \frac{1}{|h'|} \left\{ \frac{p}{4} \left| \frac{h''}{h'} \right|^2 + \epsilon \left[1 + \frac{Z}{2} \frac{h''}{h'} + \frac{\bar{Z}}{2} \frac{\bar{h}''}{\bar{h}'} \right] \right\} \ , \\ E &= \frac{|h|^2}{|h'|} \left\{ \frac{p}{4} \left| \frac{h''}{h'} - 2\frac{h'}{h} \right|^2 + \epsilon \left[1 + \frac{Z}{2} \left(\frac{h''}{h'} - 2\frac{h'}{h} \right) + \frac{\bar{Z}}{2} \left(\frac{\bar{h}''}{\bar{h}'} - 2\frac{\bar{h}'}{\bar{h}} \right) \right] \right\} \ , \\ F &= \frac{h}{|h'|} \left\{ \frac{p}{4} \left(\frac{h''}{h'} - 2\frac{h'}{h} \right) \frac{\bar{h}''}{\bar{h}'} + \epsilon \left[1 + \frac{Z}{2} \left(\frac{h''}{h'} - 2\frac{h'}{h} \right) + \frac{\bar{Z}}{2} \frac{\bar{h}''}{\bar{h}'} \right] \right\} \ , \end{aligned}$$

where

$$\begin{aligned} \mathcal{U}^\pm &= \frac{1}{\sqrt{2}} (t^\pm + z^\pm) \ , \qquad \zeta^\pm = \frac{1}{\sqrt{2}} (x^\pm + iy^\pm) \ , \\ \mathcal{V}^\pm &= \frac{1}{\sqrt{2}} (t^\pm - z^\pm) \ , \qquad \bar{\zeta}^\pm = \frac{1}{\sqrt{2}} (x^\pm - iy^\pm) \ . \end{aligned}$$

Refraction formulae of geodesics in conformally flat coordinates

Suppose C^1 geodesics $Z = Z(\tau)$, $U = U(\tau)$ and $V = V(\tau)$.
Denote the positions and velocities in the *interaction time* τ_i as

$$Z_i = Z(\tau_i)\ , \qquad U_i = U(\tau_i) = 0 \qquad V_i = V(\tau_i)\ , \qquad \dot{U}_i = \dot{U}(\tau_i)\ , \qquad \dot{V}_i = \dot{V}(\tau_i)\ , \qquad \dot{Z}_i = \dot{Z}(\tau_i)\ .$$

We will apply above two different transformations *in front of* and *behind* the impulse and we express

$$x_i^-(Z_i, V_i)\ , \qquad \dot{x}_i^-(Z_i, V_i, \dot{U}_i, \dot{V}_i, \dot{Z}_i)\ , \qquad \text{etc. for} \quad y_i^-,\ \dot{y}_i^-,\ z_i^-,\ \dot{z}_i^-,\ t_i^-,\ \dot{t}_i^-\ ,$$

and

$$Z_i(x_i^+, y_i^+, z_i^+, t_i^+)\ , \qquad V_i(x_i^+, y_i^+, z_i^+, t_i^+)\ , \qquad \dot{U}_i(x_i^+, y_i^+, z_i^+, t_i^+, \dot{x}_i^+, \dot{y}_i^+, \dot{z}_i^+, \dot{t}_i^+)\ ,$$
$$\dot{V}_i(x_i^+, y_i^+, z_i^+, t_i^+, \dot{x}_i^+, \dot{y}_i^+, \dot{z}_i^+, \dot{t}_i^+)\ , \quad \dot{Z}_i(x_i^+, y_i^+, z_i^+, t_i^+, \dot{x}_i^+, \dot{y}_i^+, \dot{z}_i^+, \dot{t}_i^+)\ .$$

It follows that:

$$\mathcal{V}_i^- = \frac{V_i}{p}\ , \qquad \dot{\mathcal{V}}_i^- = -\frac{\epsilon V_i}{p^2}(Z_i\dot{\bar{Z}}_i + \bar{Z}_i\dot{Z}_i) + \frac{\dot{V}_i}{p} - \epsilon\dot{U}_i\ ,$$
$$\mathcal{U}_i^- = \frac{Z_i\bar{Z}_i}{p}V_i\ , \qquad \dot{\mathcal{U}}_i^- = \frac{V_i}{p^2}(Z_i\dot{\bar{Z}}_i + \bar{Z}_i\dot{Z}_i) + \frac{Z_i\bar{Z}_i}{p}\dot{V}_i - \dot{U}_i\ ,$$
$$\zeta_i^- = \frac{Z_i}{p}V_i\ , \qquad \dot{\zeta}_i^- = \frac{V_i}{p^2}(\dot{Z}_i - \epsilon Z_i Z_i\dot{\bar{Z}}_i) + \frac{Z_i}{p}\dot{V}_i\ ,$$

$$h(Z_i) = \frac{\zeta_i^+}{\mathcal{V}_i^+}\ , \qquad \dot{Z}_i = \frac{p^2}{V_i}\left(\dot{\zeta}_i^+\bar{C}_{,\bar{Z}} + \dot{\bar{\zeta}}_i^+ C_{,\bar{Z}} - \dot{\mathcal{U}}_i^+ A_{,\bar{Z}} - \dot{\mathcal{V}}_i^+ B_{,\bar{Z}}\right)\ ,$$
$$V_i = \frac{\mathcal{U}_i^+}{B} = \frac{\mathcal{V}_i^+}{A} = \frac{\zeta_i^+}{C}\ , \qquad \dot{V}_i = \dot{\mathcal{U}}_i^+ D + \dot{\mathcal{V}}_i^+ E - \dot{\zeta}_i^+\bar{F} - \dot{\bar{\zeta}}_i^+ F + 2\epsilon\dot{U}_i\ ,$$
$$\dot{U}_i = \frac{1}{V_i}\left(\zeta_i^+\dot{\bar{\zeta}}_i^+ + \dot{\zeta}_i^+\bar{\zeta}_i^+ - \mathcal{U}_i^+\dot{\mathcal{V}}_i^+ - \dot{\mathcal{U}}_i^+\mathcal{V}_i^+\right)\ .$$

Now it is straightforward to write the interaction parameters behind the impulse as functions of the parameters in front the of impulse. We obtain:

- for *positions*

$$x_i^- = |h'| \frac{Z + \bar{Z}}{h + \bar{h}} x_i^+ \ , \qquad y_i^- = |h'| \frac{Z - \bar{Z}}{h - \bar{h}} y_i^+ \ ,$$

$$z_i^- = |h'| \frac{Z\bar{Z} - 1}{|h|^2 - 1} z_i^+ \ , \qquad t_i^- = |h'| \frac{Z\bar{Z} + 1}{|h|^2 + 1} t_i^+ \ , \tag{4}$$

- for *velocities*

$$\begin{aligned} \dot{x}_i^- &= a_x \dot{x}_i^+ + b_x \dot{y}_i^+ + c_x \dot{z}_i^+ + d_x \dot{t}_i^+ \ , \\ \dot{y}_i^- &= a_y \dot{x}_i^+ + b_y \dot{y}_i^+ + c_y \dot{z}_i^+ + d_y \dot{t}_i^+ \ , \\ \dot{z}_i^- &= a_z \dot{x}_i^+ + b_z \dot{y}_i^+ + c_z \dot{z}_i^+ + d_z \dot{t}_i^+ \ , \\ \dot{t}_i^- &= a_t \dot{x}_i^+ + b_t \dot{y}_i^+ + c_t \dot{z}_i^+ + d_t \dot{t}_i^+ \ , \end{aligned} \tag{5}$$

where the coefficients a, b, c, d are (complicated) functions of Z and $h(Z)$.

Refraction formulae

We may now define angles which characterize positions and velocity directions in (x, z) and (y, z) planes:

- $\alpha^{\pm}$ and $\gamma^{\pm}$ describe *position of the particle*
- $\beta^{\pm}$ and $\delta^{\pm}$ describe *inclination of the velocity vector*

$$\tan\alpha^{\pm} = \frac{x_i^{\pm}}{z_i^{\pm}}\ , \qquad \tan\gamma^{\pm} = \frac{y_i^{\pm}}{z_i^{\pm}}\ ,$$

$$\tan\beta^{\pm} = \frac{\dot{x}_i^{\pm}}{\dot{z}_i^{\pm}}\ , \qquad \tan\delta^{\pm} = \frac{\dot{y}_i^{\pm}}{\dot{z}_i^{\pm}}\ .$$

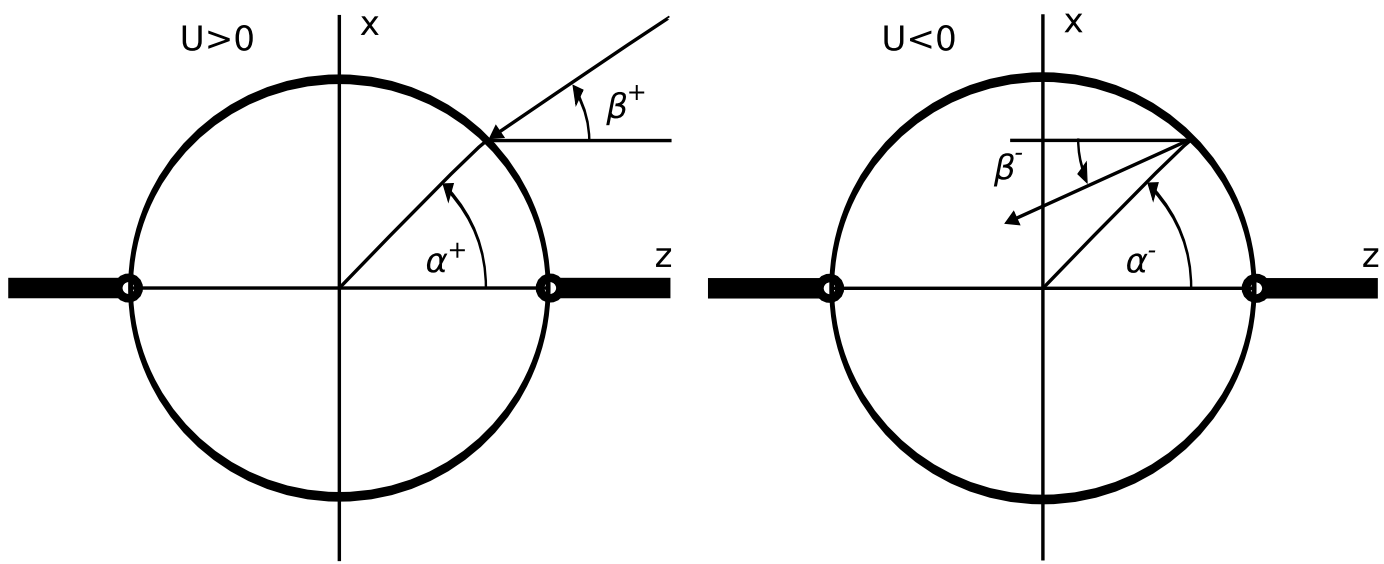


Figure 2: Geometrical meaning of angles characterizing position of the particle and inclination of its velocity in (x, z) plane. Superscript $+$ denotes quantities in front of the impulse, and $-$ behind the impulse.

Then, the formulae for positions (4) and veocities (5) can be rewritten in the form

$$\cot\alpha^- = \frac{(|Z|^2-1)\,\mathrm{Re}h}{\mathrm{Re}Z\,(|h|^2-1)}\cot\alpha^+ \ , \tag{6}$$

$$\cot\gamma^- = \frac{(|Z|^2-1)\,\mathrm{Im}h}{\mathrm{Im}Z\,(|h|^2-1)}\cot\gamma^+ \ ,$$

and

$$\tan\beta^- = \frac{v_z^+(a_x\tan\beta^+ + b_x\tan\delta^+ + c_x) + d_x}{v_z^+(a_z\tan\beta^+ + b_z\tan\delta^+ + c_z) + d_z} \ , \tag{7}$$

$$\tan\delta^- = \frac{v_z^+(a_y\tan\beta^+ + b_y\tan\delta^+ + c_y) + d_y}{v_z^+(a_z\tan\beta^+ + b_z\tan\delta^+ + c_z) + d_z} \ ,$$

where we introduced velocities with respect to the frame

$$(v_x^\pm, v_y^\pm, v_z^\pm) = \left(\frac{\dot{x}_i^\pm}{\dot{t}_i^\pm}, \frac{\dot{y}_i^\pm}{\dot{t}_i^\pm}, \frac{\dot{z}_i^\pm}{\dot{t}_i^\pm}\right) \ .$$

Example: impulsive spherical wave generated by a snapping cosmic string

In this case, the *complex mapping* $h(Z)$ is

$$h(Z) = Z^{1-\delta} \ ,$$

where δ *characterizes deficit angle* given by the presence of cosmic string outside the impulse:

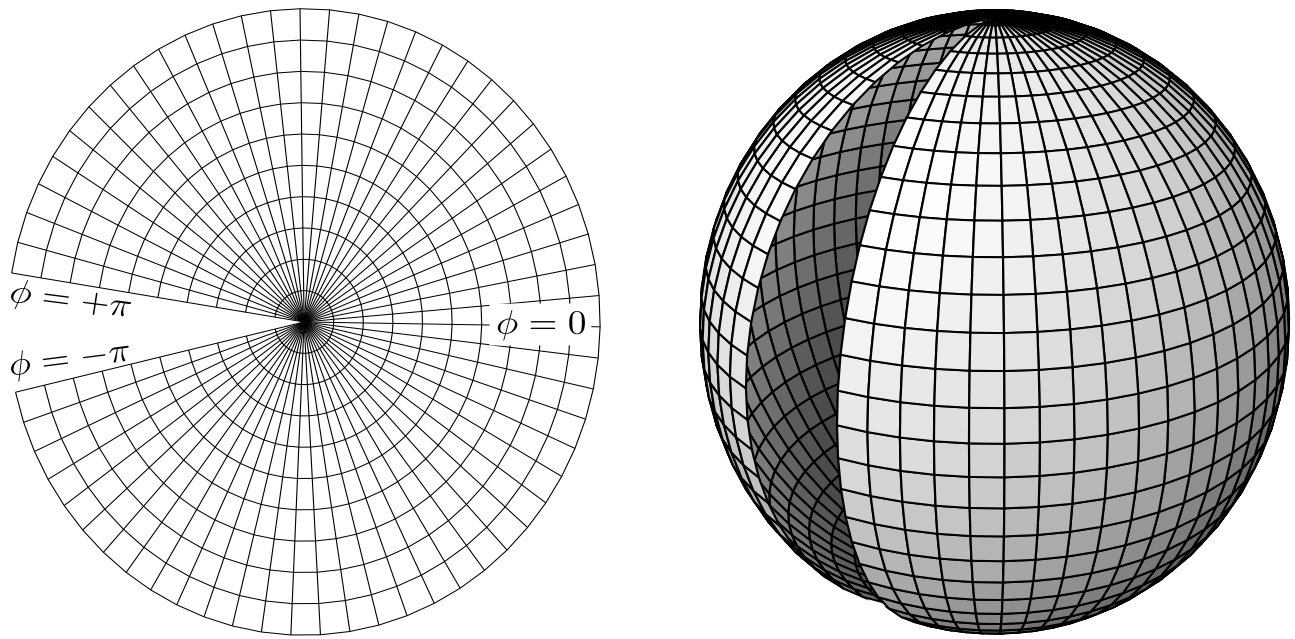


Figure 3: Mapping $Z \to h(Z) = Z^{1-\delta}$ in the complex plane (on the left) coresponds to a wedge in the Riemann sphere (on the right). The deficit angle is $2\pi\delta$.

The ring of particles standing in front of impulse

Now we can apply our general formulae (6) and (7) to the special case:

- wave is generated by the snapping string: $h(Z) = Z^{1-\delta}$
- particles are standing in (x^+, z^+) plane: $\dot{x}^+ = \dot{y}^+ = \dot{z}^+ = 0$ and $y^+ = 0$

This assumption leads to the motion only in (x, z) plane (i.e. $y^- = 0$).

For relevant coefficients in the velocity transformation (5) we obtain

$$d_x = \frac{-\delta(1-\frac{\delta}{2})}{2(1-\delta)} \left(Z^{1-\delta} + Z^{\delta-1}\right) \ ,$$
$$d_z = \frac{1}{2(1-\delta)} \left[\left(1-\tfrac{\delta}{2}\right)^2 \left(Z^{\delta} - Z^{-\delta}\right) + \tfrac{\delta^2}{4}\left(Z^{2-\delta} - Z^{\delta-2}\right)\right] \ .$$

Therefore, the changes of angles are

$$\cot\alpha^- = Z^{-\delta} \frac{Z^2 - 1}{Z^{2-2\delta} - 1} \cot\alpha^+ \ ,$$

$$\tan\beta^- = \frac{-\delta(1-\frac{\delta}{2})\left(Z^{1-\delta} + Z^{\delta-1}\right)}{\left(1-\frac{\delta}{2}\right)^2 \left(Z^{\delta} - Z^{-\delta}\right) + \frac{\delta^2}{4}\left(Z^{2-\delta} - Z^{\delta-2}\right)} \ .$$

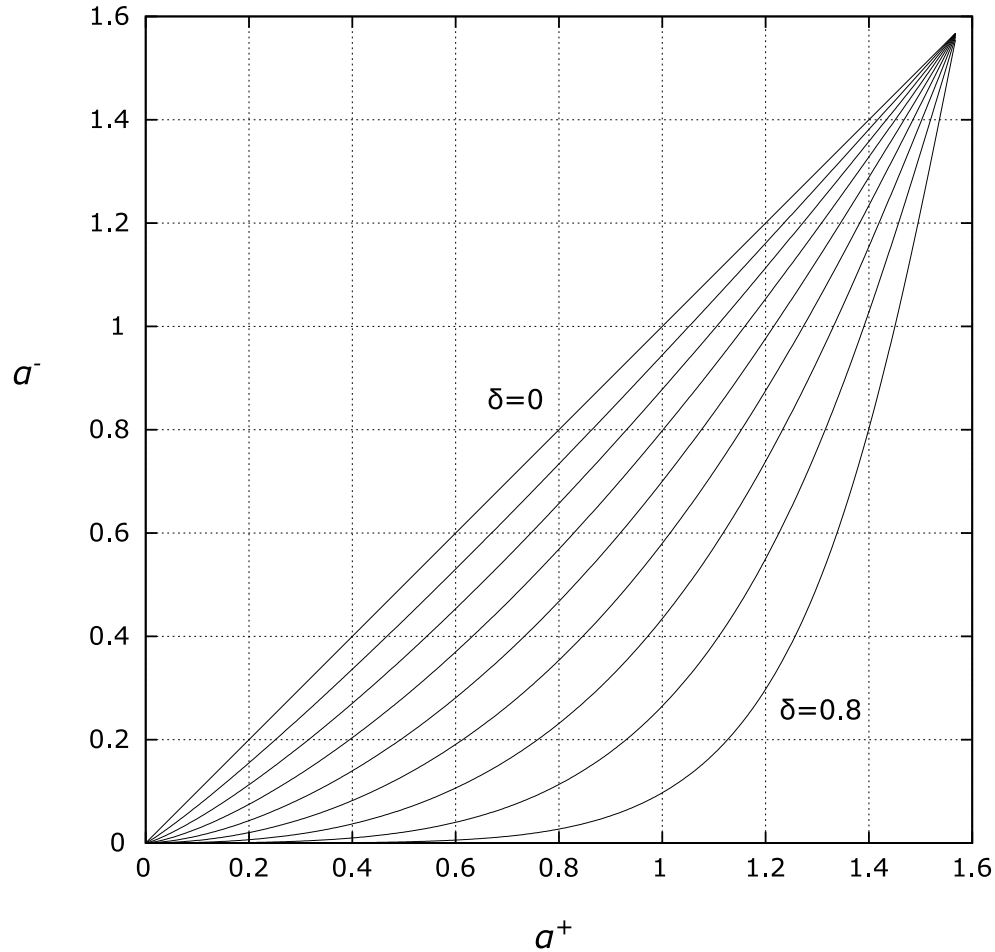


Figure 4: Shift of the position of particle induced by the wave: $\alpha^-(\alpha^+)$. The curves corespond to different values of the deficit angle: $\delta = 0,\ 0.1,\ 0.2 \ldots 0.8$.

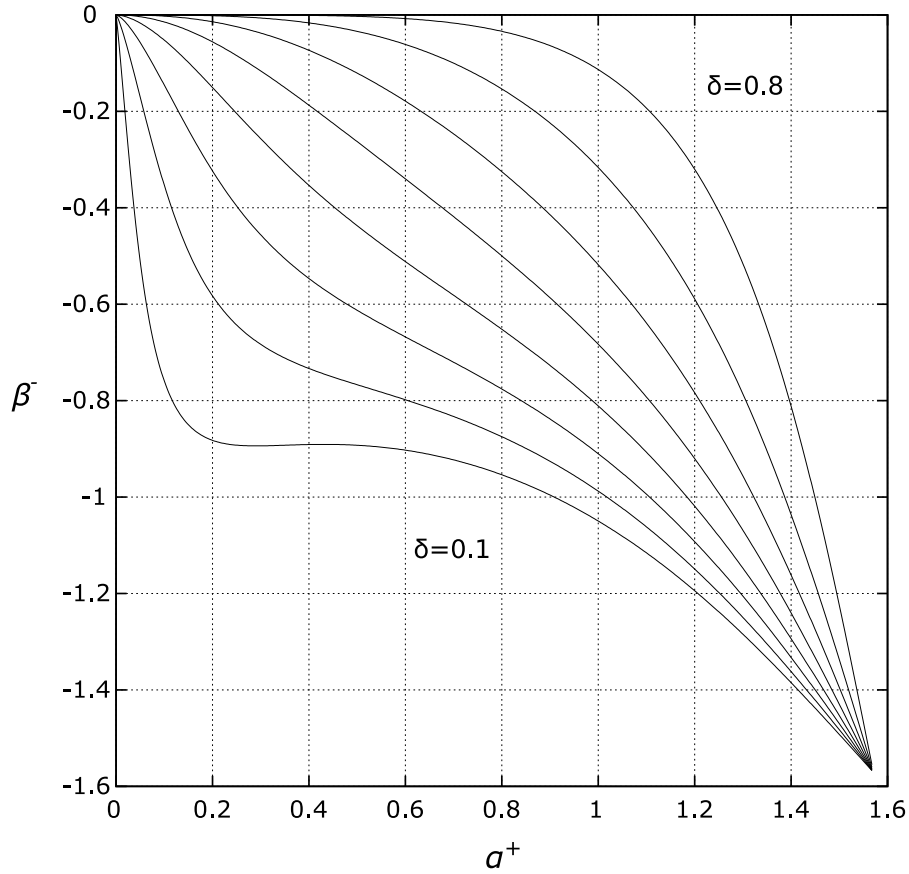


Figure 5: Dependence of velocity vector inclination on particle's position in front of impulse: $\beta^-(\alpha^+)$. The curves corespond to different values of the deficit angle: $\delta = 0.1,\ 0.2 \ldots 0.8$.

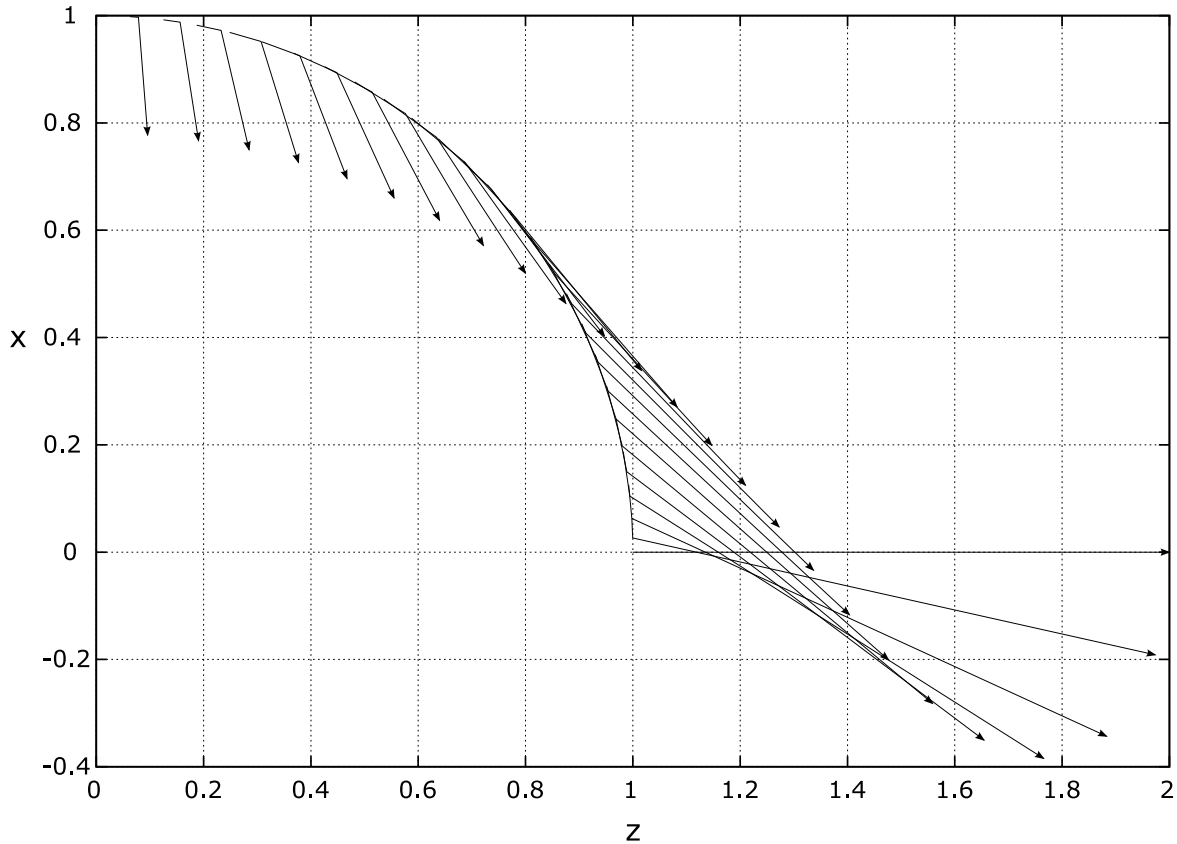


Figure 6: Shift of particle's position and change of magnitude and inclination of its velocity vector for $\delta = 0.2$.

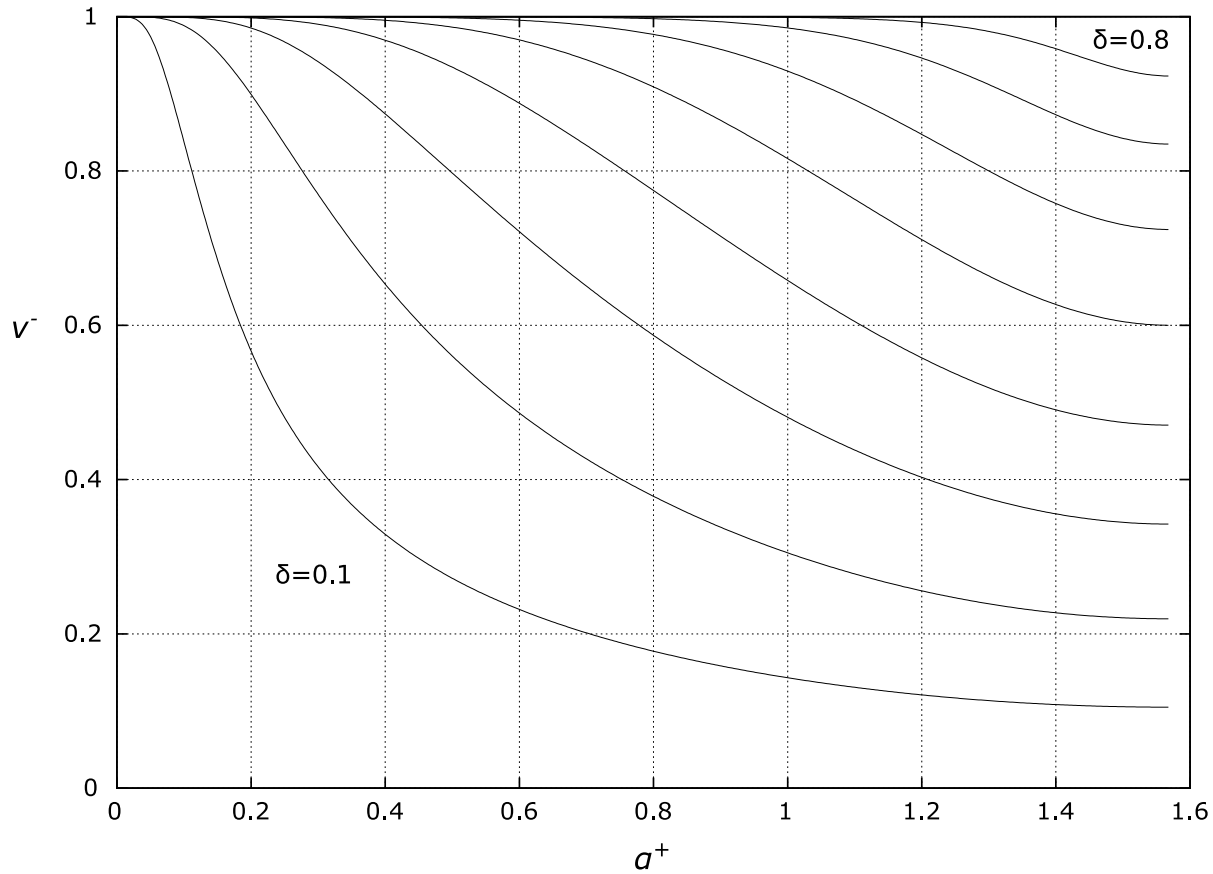


Figure 7: Velocity vector magnitude as a function of particle's position in front of impulse: $|v^-|(\alpha^+)$. The curves corespond to different values of the deficit angle: $\delta = 0.1,\ 0.2 \ldots 0.8$. The magnitude of the velocity vector is $|v^-| = \sqrt{{v_x^-}^2 + {v_z^-}^2}$.

Alternative form of the refraction formulae when $\Lambda \neq 0$

For the change of positions and velocities of test particles in 5-dimensional representation of (anti-)de Sitter spacetime we obtain

$$Z^-_{0i}=|h'|\frac{Z\bar{Z}+1}{|h|^2+1}Z^+_{0i}\ , \qquad Z^-_{1i}=|h'|\frac{Z\bar{Z}-1}{|h|^2-1}Z^+_{1i}\ ,$$

$$Z^-_{2i}=|h'|\frac{Z+\bar{Z}}{h+\bar{h}}Z^+_{2i}\ , \qquad Z^-_{3i}=|h'|\frac{Z-\bar{Z}}{h-\bar{h}}Z^+_{3i}\ ,$$

$$Z^-_{4i}=a=Z^+_{4i}\ ,$$

and

$$\begin{pmatrix} \dot{Z}^-_{2i} \\ \dot{Z}^-_{3i} \\ \dot{Z}^-_{1i} \\ \dot{Z}^-_{0i} \\ \dot{Z}^-_{4i} \end{pmatrix} = \begin{pmatrix} a_x & b_x & c_x & d_x & \omega_x \\ a_y & b_y & c_y & d_y & \omega_y \\ a_z & b_z & c_z & d_z & \omega_z \\ a_t & b_t & c_t & d_t & \omega_t \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{Z}^+_{2i} \\ \dot{Z}^+_{3i} \\ \dot{Z}^+_{1i} \\ \dot{Z}^+_{0i} \\ \dot{Z}^+_{4i} \end{pmatrix}\ ,$$

where $\omega_j = -\frac{1}{2a}\left(a_j Z^+_{2i} + b_j Z^+_{3i} + c_j Z^+_{1i} + d_j Z^+_{0i} - Z^-_{ji}\right)$ and $j = t, z, x, y$.

Example: Comoving particles in de Sitter spacetime

The global parametrization of de Sitter spacetime as an expanding 3-sphere is

$$
\begin{aligned}
Z_0 &= a \sinh \frac{t}{a} \ , \\
Z_1 &= a \cosh \frac{t}{a} \sin\chi \cos\theta \ , \\
Z_2 &= a \cosh \frac{t}{a} \sin\chi \sin\theta \cos\phi \ , \\
Z_3 &= a \cosh \frac{t}{a} \sin\chi \sin\theta \sin\phi \ , \\
Z_4 &= a \cosh \frac{t}{a} \cos\chi \ .
\end{aligned}
$$

We can apply previous equations to comoving particles in this parametrization, i. e. particles with

$$\chi^+ = const. \ , \qquad \theta^+ = const. \ , \qquad \phi^+ = const. \ ,$$

or

$$\dot{\chi}^+ = 0 \ , \qquad \dot{\theta}^+ = 0 \ , \qquad \dot{\phi}^+ = 0 \ .$$

This worldlines are timelike geodesics.

Location of the impulse

In the continuous coordinates, impulse is located on the hypersurface $U = 0$ which in the de Sitter background corresponds to

$$Z_4 = a \ , \qquad \text{resp.} \qquad {Z_1}^2 + {Z_2}^2 + {Z_3}^2 = {Z_0}^2 \ ,$$

$$\cosh \frac{t}{a} \cos \chi = 1 \ .$$

Location of cosmic string is given by the condition $Z_{23}^+ = 0$, i. e.

$$Z_{23}^+ = \frac{Z_2^+}{\cos \phi^+} = \frac{Z_3^+}{\sin \phi^+} = a \cosh \frac{t^+}{a} \sin \chi^+ \sin \theta^+ = 0 \ . \tag{8}$$

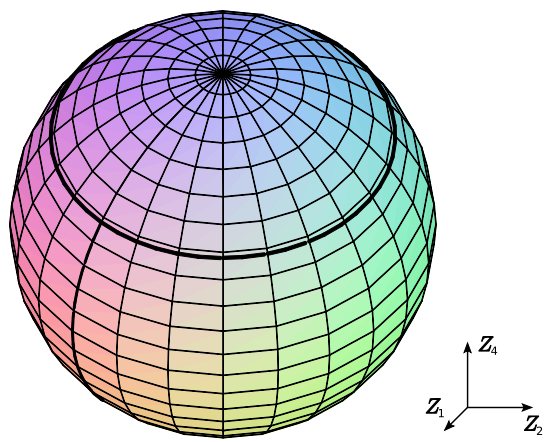


Figure 8: Cosmic string and the impulse in de Sitter space with coordinates Z_1, Z_2 a Z_4. The space is scaled on the unit sphere with ϕ suppressed. The string lies on meridians $\theta = 0, \pi$ while the impulse propagates from the north pole ($\chi = 0$) in time $t = 0$ to the equator ($\chi = \frac{\pi}{2}$) in time $t = \infty$.

Effect of the impulse on comoving particles in de Sitter space

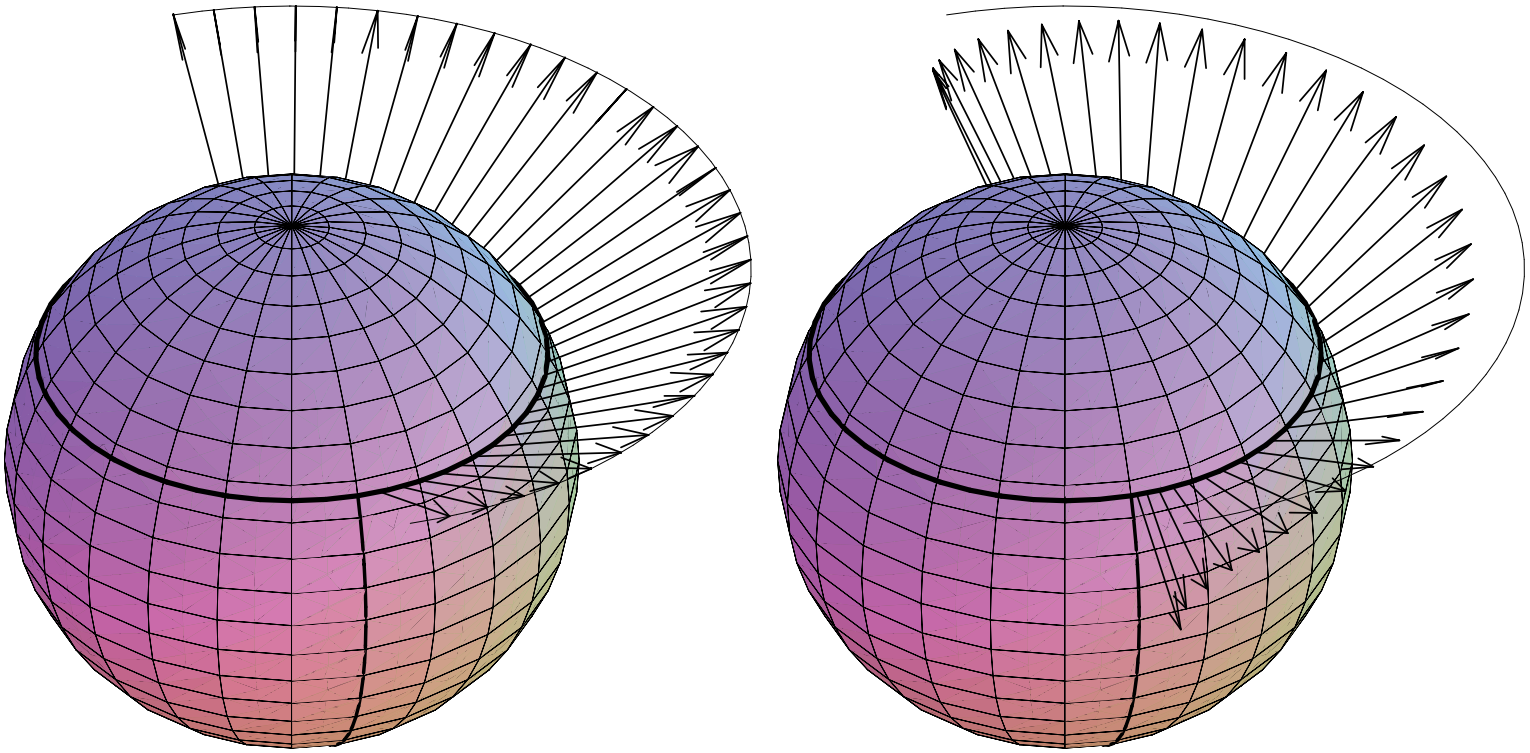


Figure 9: De Sitter space scaled on the unit sphere with 5-dimensional velocity vectors of comoving particles in front of impulse (on the left) and 5-dimensional velocity vectors of the same particles behind the impulse (on the right).

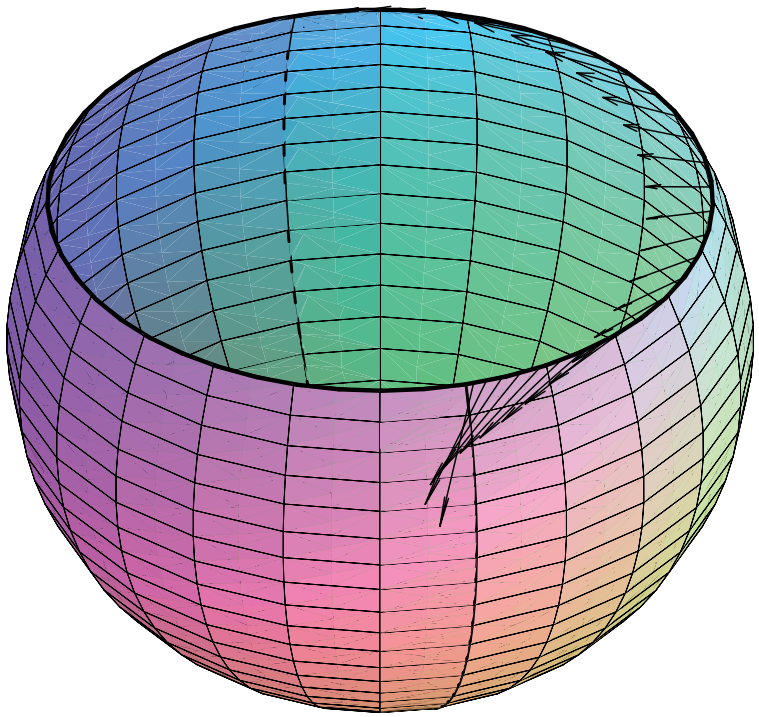


Figure 10: Velocity vectors of particles behind the impulse, with substracted comoving part in front of the impulse (de Sitter space is scaled on the unit sphere).

Conclusions

- complete description of the influence of expanding spherical impulsive gravitational waves on free test particles in spacetimes of constant curvature
- generalization previous results for Minkowski to any cosmological constant (de Sitter, anti-de Sitter universe)
- derivation of general refraction formulae describing shift of positions and change of velocity vectors of these particles
- investigation and visualization of the effect of the impulsive wave generated by a snapping cosmic string

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