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CP-VIOLATION IN NEUTRINO OSCILLATIONS AND L/E FLATNESS OF THE E-LIKE EVENT RATIO AT SUPER-KAMIOKANDE

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We show that if the presently observed L/E-flatness of the electron-like event ratio in the Super-Kamiokande atmospheric neutrino data is confirmed, then the indicated ratio must be *unity*. Further, it is found that once CP is violated the exact L/E flatness implies: (a) the CP-violating phase, in the standard parametrization, is narrowed down to two possibilities $\pm\pi/2$, and (b) the mixing between the second and the third generations must be maximal. With these results at hand, we argue that a dedicated study of the L/E-flatness of the electron-like event ratio by Super-Kamiokande can serve as an initial investigatory probe of CP violation in the neutrino sector. The assumptions under which these results hold are explicitly stated.

Keywords: Neutrino oscillations; CP-violation; atmospheric neutrinos; bi-maximal mixing.

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1. Introduction

The Super-Kamiokande data on the atmospheric neutrinos have opened a new realm of physics research.[1] The simplest interpretation of these data is flavor oscillations arising from neutrino being linear superposition of some underlying mass eigenstates. This circumstance not only takes us into the physics beyond the standard model of the high energy physics, but it also allows to probe various aspects of quantum gravity.[2–6] As such much theoretical and experimental effort is being devoted to deciphering the nature of neutrino. Here, using a very specific aspect of the

Super-Kamiokande data, we shall analytically constrain the CP-violating neutrino oscillation mixing matrix. This would help the design of future experiments, allow for more analytically-oriented theoretical research, and provide a new direction of research at the existing experimental facilities.

This work joins the on-going research with the observation that as soon as the first results from the Super-Kamiokande on atmospheric neutrinos became available, one of us emphasized that the L/E flatness noted in the *abstract* places a set of constraints on the neutrino oscillation mixing matrix.[7] However, in that, and our subsequent work,[8] CP violation has been neglected. Apart from reasons of simplicity, there is no *a priori* reason to assume the absence of CP violation in the neutrino sector. In addition, the observed cosmological baryonic asymmetry may be deeply connected with a CP violation in the leptonic sector.[9] This becomes particularly important, as we shall comment below, if the neutrino-sector CP violation is affected by gravity. As such, here, we present a nontrivial generalization of the constraints presented in the early work[7,8] to obtain a CP-violating bi-maximal matrix for neutrino oscillations.[a]

2. Analytical Constraints on the Neutrino-Oscillation Mixing Matrix

To generalize the discussion of Refs. 7 and 8, we start from the probability formula of neutrino oscillations. As in the quark sector, when neutrinos have nonzero masses, their weak eigenstates may not coincide with the mass eigenstates, but may be linear superposition of the mass eigenstates. The latter choice is precisely what is suggested by the existing data.[1,10–13] As such, in a phenomenology of neutrino oscillations, a flavor eigenstate of a neutrino is postulated to be a linear superposition of some underlying mass eigenstates

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle\,, \tag{1}$$

where $U_{\alpha j}$ is an element of the mixing matrix with $\alpha = e, \mu, \text{or } \tau$ and $j = 1, 2, 3$ in the framework of three generations. In the literature, U is usually taken as the standard parametrization matrix[14]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{2}$$

multiplied by a phase matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3+\delta_{13}} \end{pmatrix} \tag{3}$$

[a]To avoid confusion, we note in advance that in this paper we distinguish between *flux* and *event*. The former refers to the number of particles of a given species that passes a unit area in a unit time, while the latter depends on the detector sensitivity and the relevant cross-sections.

if neutrinos are of the Majorana type. Here, $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$, and ϕ_2 and ϕ_3 are the additional phases for Majorana neutrinos. Due to the unobservable effect of P in flavor oscillation experiments, we will drop it in the discussion and simply equate the mixing matrix U to V in calculations that follow. Furthermore, θ_{12}, θ_{23} and θ_{13} in U can all be made to lie in the first quadrant by an appropriate redefinition of the relevant fields.

Assuming the underlying mass eigenstates to be relativistic in the observer's frame,[15] the flavor-oscillation probability is given by[8,16]

$$P(\nu_\alpha \xrightarrow{L} \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin^2(\varphi_{jk}) + 2\sum_{j<k} \mathrm{Im}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin(2\varphi_{jk})\,, \quad (4)$$

where L, measured in meters, refers to the source-detector distance, and the flavor-oscillation inducing kinematic phases φ_{ij}, are defined as

$$\varphi_{ij} = 1.27\Delta m^2_{ij}\frac{L}{E}\,, \quad (5)$$

where E (MeV) refers to the "energy" of the flavor state, and $\Delta m^2_{ij} = m^2_i - m^2_j$, is the mass-squared difference of the underlying mass eigenstates and is measured in eV^2.

For the CP conjugate channel, the CP-odd term, i.e. the last term in Eq. (4), changes sign. So,

$$P(\bar\nu_\alpha \xrightarrow{L} \bar\nu_\beta) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin^2(\varphi_{jk}) - 2\sum_{j<k} \mathrm{Im}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin(2\varphi_{jk})\,. \quad (6)$$

Note that, all $\mathrm{Im}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})$ with $\alpha \neq \beta$ and $j \neq k$ take the same value $J_{\mathrm{CP}} = c_{12}s_{12}c_{23}s_{23}c^2_{13}s_{13}s_\delta (s_\delta = \sin\delta_{13}, c_\delta = \cos\delta_{13})$, which is the measure of CP violation.[17]

The Super-Kamiokande measured ratio, $\mathcal{R}_e$, of the electron-like events is defined as

$$\mathcal{R}_e = \frac{N'_e + N'_{\bar e}}{N_e + N_{\bar e}}\,, \quad (7)$$

where N_e and $N_{\bar e}$ are the numbers of *predicted* ν_e and $\bar\nu_e$ events in the absence of neutrino oscillations, whereas the primed quantities are the corresponding numbers of *observed* events, allowing for the presence of neutrino oscillations.

If at the top of atmosphere, i.e. the "source", the number of ν_e ($\bar\nu_e$) and ν_μ ($\bar\nu_\mu$) are $N_{\nu_e}(N_{\bar\nu_e})$ and $N_{\nu_\mu}(N_{\bar\nu_\mu})$ respectively, while the cross-sections for ν_e and $\bar\nu_e$ are σ_{ν_e} and $\sigma_{\bar\nu_e}$; then we obtain the following set of event predictions for the detector:

$$N_e = N_{\nu_e}\sigma_{\nu_e}\,, \tag{8}$$

$$N_{\bar{e}} = N_{\bar{\nu}_e}\sigma_{\bar{\nu}_e}\,, \tag{9}$$

$$N'_e = N_{\nu_e} P(\nu_e \xrightarrow{L} \nu_e)\sigma_{\nu_e} + N_{\nu_\mu} P(\nu_\mu \xrightarrow{L} \nu_e)\sigma_{\nu_e}\,, \tag{10}$$

$$N'_{\bar{e}} = N_{\bar{\nu}_e} P(\bar{\nu}_e \xrightarrow{L} \bar{\nu}_e)\sigma_{\bar{\nu}_e} + N_{\bar{\nu}_\mu} P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e)\sigma_{\bar{\nu}_e}\,. \tag{11}$$

The first two equations correspond to the absence of flavor oscillations, and the last two equations incorporate effects of flavor oscillations of neutrinos.

Now, inserting Eqs. (8)–(11) into Eq. (7), and taking note of the fact that due to CPT symmetry,

$$P(\nu_e \xrightarrow{L} \nu_e) = P(\bar{\nu}_e \xrightarrow{L} \bar{\nu}_e)\,,$$

we arrive at

$$\mathcal{R}_e - P(\nu_e \xrightarrow{L} \nu_e) = \frac{N_{\nu_\mu} P(\nu_\mu \xrightarrow{L} \nu_e)\sigma_{\nu_e} + N_{\bar{\nu}_\mu} P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e)\sigma_{\bar{\nu}_e}}{N_{\nu_e}\sigma_{\nu_e} + N_{\bar{\nu}_e}\sigma_{\bar{\nu}_e}}\,. \tag{12}$$

Finally, on defining

$$\begin{aligned} \frac{N_{\bar{\nu}_e}}{N_{\nu_e}} &= x\,, \qquad & \frac{N_{\bar{\nu}_\mu}}{N_{\nu_\mu}} &= y\,, \\ \frac{\sigma_{\bar{\nu}_e}}{\sigma_{\nu_e}} &= \lambda\,, \qquad & \frac{N_{\nu_\mu}}{N_{\nu_e}} &= r\,, \end{aligned} \tag{13}$$

it is easy to show that

$$\mathcal{R}_e - P(\nu_e \xrightarrow{L} \nu_e) = \frac{r}{1+\lambda x}\left(P(\nu_\mu \xrightarrow{L} \nu_e) + \lambda y P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e)\right). \tag{14}$$

Now, substituting Eqs. (4) and (6) into the above equation, and after simplifying, we obtain

$$\begin{aligned} &\left\{|U_{e1}|^2|U_{e2}|^2 + r\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 1}U_{e1}^*U_{\mu 2}^*U_{e2})\right\}\sin^2(\varphi_{12}) \\ &\quad + \left\{|U_{e1}|^2|U_{e3}|^2 + r\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 1}U_{e1}^*U_{\mu 3}^*U_{e3})\right\}\sin^2(\varphi_{13}) \\ &\quad + \left\{|U_{e2}|^2|U_{e3}|^2 + r\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 2}U_{e2}^*U_{\mu 3}^*U_{e3})\right\}\sin^2(\varphi_{23}) \\ &\quad - \frac{r}{2}\frac{1-\lambda y}{1+\lambda x}J_{\mathrm{CP}}[\sin(2\varphi_{12}) + \sin(2\varphi_{13}) + \sin(2\varphi_{23})] \\ &\quad = \frac{1}{4}(1-\mathcal{R}_e)\,. \end{aligned} \tag{15}$$

It is worth noting that in case $x = y$ and $J_{\mathrm{CP}} = 0$, i.e. if the ratio of the numbers of $\bar{\nu}_e$ to ν_e equals that of the numbers of $\bar{\nu}_\mu$ to ν_μ at the source, and if there is no

CP violation in the neutrino sector, Eq. (15) loses dependence on the neutrino and anti-neutrino cross-sections.

In order that Eq. (15) holds for all values of L/E we must impose the constraints[b]:

$$\frac{r}{2}\frac{1-\lambda y}{1+\lambda x} J_{\rm CP} = 0 \tag{16}$$

and

$$|U_{e1}|^2|U_{e2}|^2 + r\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 1}U_{e1}^* U_{\mu 2}^* U_{e2}) = 0\,, \tag{17}$$

$$|U_{e1}|^2|U_{e3}|^2 + r\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 1}U_{e1}^* U_{\mu 3}^* U_{e3}) = 0\,, \tag{18}$$

$$|U_{e2}|^2|U_{e3}|^2 + r\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 2}U_{e2}^* U_{\mu 3}^* U_{e3}) = 0\,. \tag{19}$$

3. The Constrained CP-Violating Matrix

Since Eq. (15) holds for any value of L/E, we are also free to set $L/E = 0$. This yields:

$$\mathcal{R}_e = 1\,. \tag{20}$$

Although we invoke the Super-Kamiokande observed flatness for $\mathcal{R}_e$ from the beginning, we did *not* refer to a specific value of $\mathcal{R}_e$. The present analysis *predicts* $\mathcal{R}_e$ to be unity. This circumstance is in sharp contrast to the framework of Refs. 7 and 8 where one assumes both the indicated flatness and the value unity for $\mathcal{R}_e$.

Furthermore, Eq. (16) requires that $J_{\rm CP} = 0$ and/or $\lambda y = 1$. We consider each of these cases in turn.

3.1. $J_{\rm CP} = 0$ *case*

The constraints (16)–(19), after some algebraic manipulations, reduce to:

$$c_{12}s_{12}c_{13}^2 + r\frac{1+\lambda y}{1+\lambda x}\{c_{12}s_{12}(s_{23}^2 s_{13}^2 - c_{23}^2) + (s_{12}^2 - c_{12}^2)c_{23}s_{23}s_{13}\} = 0\,, \tag{21}$$

$$c_{12}s_{13} - r\frac{1+\lambda y}{1+\lambda x}s_{23}(c_{12}s_{23}s_{13} + s_{12}c_{23}) = 0\,, \tag{22}$$

$$s_{12}s_{13} - r\frac{1+\lambda y}{1+\lambda x}s_{23}(s_{12}s_{23}s_{13} - c_{12}c_{23}) = 0\,. \tag{23}$$

We find no nontrivial solution that satisfies this set of equations. However, a limit of the second case to be considered next does yield a non-CP-violating mixing matrix and reproduces the results given in Ref. 8.

[b] The Super-Kamiokande data span roughly five orders of magnitude in L/E. However, as a mathematical theorem, it can be shown that if $\mathcal{R}_e$ carries an L/E independence over a finite range of L/E, then it must be so over the entire range of L/E.

3.2. $\lambda y = 1$ case

According to the definition, $\lambda y = 1$ indicates that, if the ratio of the numbers of ν_μ to $\bar{\nu}_\mu$ is close to that of the cross-sections of $\bar{\nu}_e$ to ν_e, then this circumstance allows to ignore the last term on the left-hand side of Eq. (15). From Table 1 of Ref. 18 we estimate $y \approx 2.06 \pm 0.31$,[c] while from Ref. 19 we infer $\lambda \approx 1/2.4$. Thus, the required condition is fulfilled on "accidental" grounds. Further justification for ignoring the indicated term lies in the fact that J_{CP} is significantly suppressed by data-indicated $U_{e3} \ll 1$. In any case E-dependent deviations from $\lambda y = 1$ would contribute to departures from the exact L/E flatness of the e-like event ratio. Similarly, we point out that in certain range of L/E the matter effects may become operative, and these too would contribute to the indicated departure.

Substituting the relevant elements of U into Eqs. (17)–(19), similarly, we obtain

$$c_{12}s_{12}c_{13}^2 + \frac{2r}{1+\lambda x}\{c_{12}s_{12}(s_{23}^2 s_{13}^2 - c_{23}^2) + (s_{12}^2 - c_{12}^2)c_{23}s_{23}s_{13}c_\delta\} = 0\,, \qquad (24)$$

$$c_{12}s_{13} - \frac{2r}{1+\lambda x}s_{23}(c_{12}s_{23}s_{13} + s_{12}c_{23}c_\delta) = 0\,, \qquad (25)$$

$$s_{12}s_{13} - \frac{2r}{1+\lambda x}s_{23}(s_{12}s_{23}s_{13} - c_{12}c_{23}c_\delta) = 0\,. \qquad (26)$$

From Eqs. (24) and (25) we infer,

$$s_{23}^2 = \frac{1+\lambda x}{2r} \qquad (27)$$

and

$$c_\delta = 0\,. \qquad (28)$$

So, the CP phase is $\pi/2$ or $-\pi/2$. Inserting Eqs. (27) and (28) into Eq. (24), we have

$$c_{23}^2 = \frac{1+\lambda x}{2r}\,. \qquad (29)$$

Finally, combining Eqs. (27) and (29), we achieve the results:

$$\theta_{23} = \pi/4\,, \qquad r = 1 + \lambda x\,. \qquad (30)$$

That is the mixing between the second and the third generations is maximal, and that the ratio of the numbers of ν_μ to ν_e equals to one plus the ratio of the numbers of $\bar{\nu}_e$ to ν_e events in case of no oscillations.

As a result, the indicated L/E flatness in the the Super-Kamiokande data on the atmospheric neutrinos implies CP-violating maximal mixing matrix:

[c] It being the value associated with the lowest atmospheric density in the experiment, identified here as "the top of the atmosphere".

$$U^{\pm} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & \mp i s_{13} \\ -\frac{1}{\sqrt{2}}(s_{12} \pm i c_{12} s_{13}) & \frac{1}{\sqrt{2}}(c_{12} \mp i s_{12} s_{13}) & \frac{1}{\sqrt{2}} c_{13} \\ \frac{1}{\sqrt{2}}(s_{12} \mp i c_{12} s_{13}) & -\frac{1}{\sqrt{2}}(c_{12} \pm i s_{12} s_{13}) & \frac{1}{\sqrt{2}} c_{13} \end{pmatrix}, \quad (31)$$

where U^+ corresponds to $\delta_{13} = \pi/2$ and U^- arises from $\delta_{13} = -\pi/2$.

4. Concluding Remarks

Corresponding to the two general forms for U, we obtain the following two measures of CP-violation:

$$J_{\mathrm{CP}}^{\pm} = \pm\frac{1}{2} c_{12} s_{12} c_{13}^2 s_{13} = \pm\frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{13}) \cos(\theta_{13}) . \quad (32)$$

In the limit θ_{13} vanishes, the $U^{\pm}$ reduces to the result contained in Eq. (26) of Ref. 8, as it should. Preliminary indications that the U matrix carries a similar form as given in Eq. (31) can also be deciphered from a recent work of Barger, Geer, Raja, and Whisnant.[20] Furthermore, for $\theta_{12} = \pi/4$, U^+ reads

$$U^{+} = \begin{pmatrix} c_{13}/\sqrt{2} & c_{13}/\sqrt{2} & -i s_{13} \\ -(1 + i s_{13})/2 & (1 - i s_{13})/2 & c_{13}/\sqrt{2} \\ (1 - i s_{13})/2 & -(1 + i s_{13})/2 & c_{13}/\sqrt{2} \end{pmatrix} \quad (33)$$

which coincides with the Xing postulate.[21] The latter, originally invoked to simultaneously allow for the a neutrino-oscillation explanation of the atmospheric and solar neutrino data, turns out to be dictated upon us by the indicated L/E flatness.

Since the CHOOZ experiment[22] constraints, for large-δm^2, $\sin^2(2\theta_{13})$ to be about 0.1, even the large value of $\delta_{13} = \pm\pi/2$ implied by the present analysis, does not result in a maximal CP-violating difference:

$$P(\nu_\alpha \xrightarrow{L} \nu_\beta) - P(\bar{\nu}_\alpha \xrightarrow{L} \bar{\nu}_\beta) = 4 J_{\mathrm{CP}} \sum_{j<k} \sin(2\varphi_{jk}) . \quad (34)$$

However, we note that Eqs. (4) and (6) define a set of flavor-oscillation clocks, and these clocks must redshift when introduced in a gravitational environment. If this environment is characterized by a dimensionless gravitational potential, Φ_{grav}, then in order that the flavor-oscillations suffer a gravitationally-induced redshift we must replace, in Eq. (34), φ_{jk} by $(1 + \Phi_{\mathrm{grav}})\varphi_{jk}$. For other quantum-gravity effects on neutrino oscillations we refer the reader to Ref. 19. Such gravitationally-induced modifications to a neutrino-sector CP-violation may carry significant physical implications.

5. Summary

In summary, firstly, our discussion extended in this work seems to obligate us to accept a CP-violated neutrino sector. And secondly, once CP is violated in neutrino system, the exact L/E flatness of $\mathcal{R}_e$ implies that: (i) the mixing between the second and the third generations must be maximal, (ii) the ratio $\mathcal{R}_e$ must be unity, (iii) the CP-violating phase in the standard parametrization matrix is $\pi/2$ up to a sign ambiguity, (iv) $N_{\nu_\mu}\sigma_{\nu_e} = N_{\bar{\nu}_\mu}\sigma_{\bar{\nu}_e}$, and finally that (v) $N_{\nu_\mu}/N_{\nu_e} = 1 + N_{\bar{\nu}_e}\sigma_{\bar{\nu}_e}/N_{\nu_e}\sigma_{\nu_e}$.

Therefore, a dedicated study of the ratio $\mathcal{R}_e$ in terms of its precise value, and its L/E dependence, can become a powerful probe to study CP-violation in the neutrino sector. Within the framework of this study, if the future data confirms $\mathcal{R}_e$ to be unity for all zenith angles, then we must conclude that either there is no CP-violation in the neutrino sector, or it is of the form predicted by Eq. (32). This precise result, in conjunction with knowledge of θ_{12}, θ_{13}, and the associated mass-squared differences, up to a sign ambiguity, completely determines the expectations for CP-violation in all neutrino-oscillation channels.

However, the assumptions made in arriving the above results may be violated to some extent, and we once again point out that the E-dependent deviations from $\lambda y = 1$ would contribute to departures from the exact L/E flatness of the e-like event ratio. Similarly, we note that in certain range of L/E the matter effects may become operative, and these too would contribute to the indicated departure. Once deviations from $\lambda y = 1$ are fully incorporated, the study of the L/E flatness of the e-like event ratio at Super-Kamiokande probes not only CP-violation in the neutrino sector, but it also explores absence/presence of matter effects in atmospheric neutrino oscillations. At present the available data contains significant systematic and statistical errors, and, for that reason, these higher order corrections are left to a future investigation.

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