Empirical Neutrino Mass Matrix Related to Up-Quark Masses

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A new approach to quark and lepton masses is proposed: the mass spectra originate in vacuum expectation values of U(3) flavor nonet (gauge singlet) scalars. For example, we assume a superpotential \( W = W_Y + W_e + W_u + W_d \), where

\[
W_Y = \sum_{i,j} \frac{y_e}{\Lambda} (Y_e)_j^i L_i H_u N_j + \sum_{i,j} \frac{y_e}{\Lambda} (Y_e)_j^i L_i H_d E_j + \cdots ,
\]

(1)

\[
W_e = \lambda_e \text{Tr}[\Phi_e \Phi_e \Phi_e] + m_e \text{Tr}[\Phi_e \Phi_e] + \mu_e^2 \text{Tr}[\Phi_e] + \lambda_Y e \text{Tr}[\Phi_e Y_e Y_e] + m_{Y_e} \text{Tr}[Y_e Y_e],
\]

(2)

and so on. From SUSY vacuum conditions \( \partial W / \partial \Phi_e = 0 \) and \( \partial W / \partial Y_e = 0 \), we obtain a bilinear charged lepton mass relation

\[
Y_e = -(\lambda_Y e / 2m_{Y_e}) \Phi_e Y_e + 3 \lambda_e \Phi_e + 2m_{Y_e} + \mu_e^2 \mathbf{1} + \lambda_Y e (Y_e Y_e + Y_e Y_e) = 0,
\]

respectively. The latter equation together with the former one leads to a cubic equation

\[
(c_3^e \Phi_e + c_2^e \Phi_e + c_1^e \Phi_e + c_0^e \mathbf{1}) = 0,
\]

which can completely determine three eigenvalues of \( \langle \Phi_e \rangle \), where \( c_3^e = \lambda_Y e / m_{Y_e} \), \( c_2^e = -3 \lambda_e \), \( c_1^e = -2m_e \) and \( c_0^e = -\mu_e^2 \), so that, for example, we obtain

\[
\frac{m_e + m_\mu + m_\tau}{\sqrt{m_e + m_\mu + m_\tau}} = \frac{\text{Tr}[\Phi_e \Phi_e]}{\text{Tr}^2[\Phi_e]} = 1 - 2 \frac{c_2^e c_3^e}{(c_2^e)^2},
\]

(3)

\[
\frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e + m_\mu + m_\tau})^3} = \frac{\det \Phi_e}{\text{Tr}^3[\Phi_e]} = \frac{c_0^e (c_3^e)^2}{(c_2^e)^3}.
\]

(4)

Thus, in the new approach [1], we investigate relations among quark and lepton masses and mixings, not by treating the Yukawa coupling constants directly, but by investigating the structure of the superpotential.

In the present work [2], we will propose a specific form of \( W_e = W_e(Y_e, \Phi_e, Y_e) \) in addition to \( W_e = W_e(Y_e, \Phi_e) \) (Eq.(2)) and \( W_u = W_u(Y_u, \Phi_u) \) (similar to Eq.(2)) from phenomenological consideration, and thereby we will find an empirical neutrino mass matrix which is described in terms of the up-quark and charged lepton masses and which leads to a nearly tribimaximal mixing without assuming any discrete symmetry:

\[
(M^\text{Dirac})_{ij} = m_0^\nu \left[ \frac{1}{m_{ee}} + \frac{1}{m_{e\mu}} \right] [V(\pi) \Phi_u^D V(\pi)]_{ij} + \xi_0 \delta_{ij},
\]

(5)

where \( (\Phi_u^D)_{ii} \propto \sqrt{m_{ui}} \) and \( V(\delta) \) with \( \delta \simeq \pi/3 \) is the observed CKM matrix. Note that if \( Y_d \) and \( Y_e \) are simultaneously diagonalized in a same basis, the matrix form of \( \Phi_u (\propto Y_u^{1/2}) \) will be described by \( V(\pi/3) \Phi_u^D V(\pi/3) \) on a diagonal basis of \( Y_d \). When we use \( V(\pi) \) instead of \( V(\pi/3) \), the matrix (5) can predict \( \sin^2 2\theta_{23} = 1.00 \) and \( \tan^2 \theta_{12} = 0.513 \) from the observed up-quark and charged lepton masses and CKM matrix parameters, independently of the parameter \( \xi_0 \).
