Empirical Neutrino Mass Matrix Related to Up-Quark Masses

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A new approach to quark and lepton masses is proposed: the mass spectra originate in vacuum expectation values of U(3) flavor nonet (gauge singlet) scalars. For example, we assume a superpotential $W = W_Y + W_e + W_u + W_\nu + W_d$, where

$$W_Y = \sum_{i,j} \frac{y_\nu}{\Lambda}(Y_\nu)_j^i L_i H_u N^j + \sum_{i,j} \frac{y_e}{\Lambda}(Y_e)_j^i L_i H_d E^j + \cdots , \tag{1}$$

$$W_e = \lambda_e \mathrm{Tr}[\Phi_e \Phi_e \Phi_e] + m_e \mathrm{Tr}[\Phi_e \Phi_e] + \mu_e^2 \mathrm{Tr}[\Phi_e] + \lambda_{Ye} \mathrm{Tr}[\Phi_e \Phi_e Y_e] + m_{Ye} \mathrm{Tr}[Y_e Y_e], \tag{2}$$

and so on. From SUSY vacuum conditions $\partial W/\partial \Phi_e = 0$ and $\partial W/\partial Y_e = 0$, we obtain a bilinear charged lepton mass relation $Y_e = -(\lambda_{Ye}/2m_{Ye})\Phi_e\Phi_e$, and $3\lambda_e\Phi_e\Phi_e + 2m_e\Phi_e + \mu_e^2\mathbf{1} + \lambda_{Ye}(\Phi_e Y_e + Y_e\Phi_e) = 0$, respectively. The latter equation together with the former one leads to a cubic equation $c_3^e\Phi_e\Phi_e\Phi_e + c_2^e\Phi_e\Phi_e + c_1^e\Phi_e + c_0^e\mathbf{1} = 0$, which can completely determine three eigenvalues of $\langle\Phi_e\rangle$, where $c_3^e = \lambda_{Ye}^2/m_{Ye}$, $c_2^e = -3\lambda_e$, $c_1^e = -2m_e$ and $c_0^e = -\mu_e^2$, so that, for example, we obtain

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{\mathrm{Tr}[\Phi_e\Phi_e]}{\mathrm{Tr}^2[\Phi_e]} = 1 - 2\frac{c_1^e c_3^e}{(c_2^e)^2}, \tag{3}$$

$$\frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{\det\Phi_e}{\mathrm{Tr}^3[\Phi_e]} = \frac{c_0^e (c_3^e)^2}{(c_2^e)^3}. \tag{4}$$

Thus, in the new approach [1], we investigate relations among quark and lepton masses and mixings, not by treating the Yukawa coupling constants directly, but by investigating the structure of the superpotential.

In the present work [2], we will propose a specific form of $W_\nu = W_\nu(Y_\nu, \Phi_u, Y_e)$ in addition to $W_e = W_e(Y_e, \Phi_e)$ (Eq.(2))and $W_u = W_u(Y_u, \Phi_u)$ (similar to Eq.(2)) from phenomenological consideration, and thereby we will find an empirical neutrino mass matrix which is described in terms of the up-quark and charged lepton masses and which leads to a nearly tribimaximal mixing without assuming any discrete symmetry:

$$(M_\nu^{Dirac})_{ij} = m_0^\nu \left[\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right) [V^\dagger(\pi)\Phi_u^D V(\pi)]_{ij} + \xi_0 \delta_{ij} \right], \tag{5}$$

where $(\Phi_u^D)_{ii} \propto \sqrt{m_{ui}}$ and $V(\delta)$ with $\delta \simeq \pi/3$ is the observed CKM matrix. Note that if Y_d and Y_e are simultaneously diagonalized in a same basis, the matrix form of Φ_u ($\propto Y_u^{1/2}$) will be described by $V^\dagger(\pi/3)\Phi_u^D V(\pi/3)$ on a diagonal basis of Y_d. When we use $V(\pi)$ instead of $V(\pi/3)$, the matrix (5) can predict $\sin^2 2\theta_{23} = 1.000$ and $\tan^2\theta_{12} = 0.513$ from the observed up-quark and charged lepton masses and CKM matrix parameters, independently of the parameter ξ_0.

[1] Y. Koide, arXive:0802.1082, to be published in Phys.Lett. B (2008).

[2] Y. Koide, in preparation.